



# RAMC 2022

## Elementary II Team Solutions

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*Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:*

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1. Natalia is opening a book shop. She buys 300 books, and decides that she will sell each book for \$5 more than she bought it for. If she sells all 300 books, what will her profit be, in dollars?

**Answer:**  $\boxed{1500}$

**Solution:** The formula for profit is Sales revenue - cost. Since Natalia is selling each book for \$5 more than she bought them, she will earn \$5 on each book. She has 300 books so she will earn  $\$5 \cdot 300 = \$1500$ .

2. Jimmy is starting a new job. The starting pay is \$40,000 per year, but his boss said that next year his salary will increase by 10%. What will his salary be next year, in dollars?

**Answer:**  $\boxed{\$44000}$

**Solution:** If his job salary will increase by 10%, we add 10% of \$40,000 to the original salary.

$$\$40000 + 10\% \cdot \$40000 = \$40000 + .1 \cdot \$40000 = 1.1 \cdot \$40000 = \$44000.$$

3. What is the sum of the mean, median, and mode of the numbers 3, 4, 1, 4, 2, 5, 1, 4, 12?

**Answer:**  $\boxed{12}$

**Solution:** Lets rearrange the numbers into increasing order.

1 1 2 3 4 4 4 5 12

The mean is equal to  $\frac{1+1+2+3+4+4+4+5+12}{9} = \frac{36}{9} = 4$ . The median is the 5<sup>th</sup> number, which is also 4. The mode is also 4, as it appears the most. Therefore, the sum of the mean, median, and mode of the numbers is  $4 + 4 + 4 = 12$ .

4. Vanessa has a square of area 25. Four equal sized 3-4-5 right triangles are attached to the exterior of the square, with the hypotenuse being attached to the square. What is the total area of all 5 shapes added together?

**Answer:**  $\boxed{49}$

**Solution:** A square with area 25 results in each side being length 5. This means the hypotenuse of the attached triangles also have length 5. Using a 3 - 4 - 5 right triangle, the remaining two sides are 3 and 4. Each triangle's area is  $\frac{3 \cdot 4}{2} = 6$ . As there are 4 triangles and one square, we have our total area is  $24 + 25 = 49$ .

5. Beth's Card-N-Dice club has 100 members, some whom enjoy cards, some enjoy dice, and some enjoy both. The manager calculated that if 10 only-card enthusiasts changed to be only-dice enjoyers, there would be 3 times as many members who only like dice when compared to those who only enjoyed cards. Let  $x$  be the number of members who like dice, and  $y$  be the number of members who like cards. Given that there are 20 enjoyers for both card and dice, find  $xy$ .

**Answer:**

**Solution:** We note that as there are 20 members who enjoy both, there are  $100 - 20 = 80$  members who like only cards or only dice. As the manager calculated, it would be a 1 : 3 ratio if 10 only-card members moved to be dice. This 1 : 3 ratio is 20 : 60 with 80 members, meaning that before the switch, there would be  $20 + 10 = 30$  only-card members, and  $60 - 10 = 50$  only-dice members. As we have 20 who like both,  $x = 30 + 20 = 50$ , and  $y = 50 + 20 = 70$ . Therefore,  $xy = 50 \cdot 70 = 3500$ .

6. Every morning, Tina has 180 choices of an outfit combination, which is made up of a colored shirt, pants, and a pair of shoes. Let the number of unique pants she has be  $x$ , and the number of different pairs of shoes be  $y$ . If the number of different colored shirts she has is 6, find the maximum value of  $x + y$ .

**Answer:**

**Solution:** The number of outfits Tina has can be calculated by  $6xy$ , as Tina chooses 1 shirt from 6 (6 choices), 1 pant from  $x$ , and 1 pair of shoes from  $y$ . Therefore,  $xy = \frac{180}{6} = 30$ . As we need to find the maximum value of  $xy$ , we find by guess and check, or by maximizing the value of  $x + \frac{30}{x}$ , to find that our optimal value of  $(x, y) = (1, 30)$  or  $(30, 1)$ . Therefore, the maximum value of  $x + y$  is 31.

7. There exists a completely flat and straight train track, and on it lies 2 train stations; one is 1 km west of the other. The train from the western train station leaves at 2:25 pm for a city a long ways away, traveling west at 22 km/h. Another train leaves at 4:25 pm from the eastern train station, again traveling to a distance far away, heading east at 40 km/h. At what time will both trains be the same distance from the eastern train station? Express your answer in the form hour:minute, and include if it is am or pm.

**Answer:** 6:55 pm

**Solution:** We can write the distance the west train is from the east train station to be  $22h + 1$ , where  $h$  = the number of hours past 2:25pm. Similarly, the distance that the east train is from the east train station is  $40(h - 2)$ . So, we set those 2 equal to each other.

$$\begin{aligned}22h + 1 &= 40(h - 2) \\ &= 40h - 80 \\ 18h &= 81 \\ h &= \frac{9}{2}\end{aligned}$$

Therefore, the time they are equidistant from the right train station is 4.5 hours after 2:25pm, which is at 6:55pm.

8. Water flows into a swimming pool 3 times faster than it leaks out of a swimming pool. If the pool starts with no water, and after 30 minutes, there is 60 liters of water. How many liters of water leak out of the swimming pool every minute?

**Answer:** 1

**Solution:** We can calculate the net gain of the water to be 60L/30minutes, which results in a 2L gain every minute. Knowing that the ratio between water in and out is a 3:1 ratio, we divide the 2L by 2, leaving the rate at which water leaks out to be at 1L/minute.

9. A magic rocket is launched from a platform 15 meters above the ground. It is launched directly straight up into the air, and its speed stays constant at a rate of 167 meters per second. If the magic rocket is immune to the effects of gravity, how far is the rocket from the platform after 17 seconds, in meters?

**Answer:** 2839

**Solution:** After 17 seconds, the rocket will move  $167 \cdot 17 = 2839$  meters into the air. As it is launched straight up, there is only the need to calculate height.

10. At the St. Dairy's Hospital, there are 10 medical support staff. Each member on staff wears either a regular surgical mask or a K95 mask. There are 3 support staff using a K95 mask. For any operation, three of the ten support staff are chosen to help, at random. What is the probability that any operation has at least 1 staff member wearing a surgical mask?

**Answer:**  $\frac{119}{120}$

**Solution:** We note that  $P(\text{At least 1 surgical}) = 1 - P(\text{None surgical})$ .

The probability that none are wearing a surgical mask can be calculated by finding the number of ways to find three members that wear a K95 over the total amount of three-member teams. As there are only 3 members who have a K95, there is only 1 possible team. The total amount of teams is  $\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$ . Therefore,  $P(\text{None surgical}) = \frac{1}{120}$ , and  $P(\text{At least 1 surgical}) = 1 - \frac{1}{120} = \frac{119}{120}$ .