RAMC 2022

Elementary II Tiebreaker Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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1. Maggie has a 3 by 3 grid with 5 shaded tiles. Zoey has an 6 by 6 grid, but also has 5 shaded tiles. Jenny needs to guess which square is shaded on both boards. Let the probability that she guesses wrong, then right, on the 3 by 3 board be x, and on the 6 by 6 board be y, respectively. If Jenny can guess the same tile twice, find $x - y$.

   Answer: $\boxed{\frac{55}{432}}$

   Solution: On the 3×3 board, Jenny has a probability of

   $$x = \frac{4}{9} \cdot \frac{5}{9} = \frac{20}{81} = \frac{320}{1296}.$$

   On the 6×6 board, Jenny has a probability of

   $$y = \frac{31}{36} \cdot \frac{5}{36} = \frac{155}{1296}.$$

   Therefore, $x - y = \frac{320}{1296} - \frac{155}{1296} = \frac{165}{1296} = \frac{55}{432}$.

2. What is the sum of the sum of the distinct prime factors of 2022?

   Answer: $\boxed{342}$

   Solution: The prime factorization is $2022 = 2 \cdot 3 \cdot 337$, which adds up to $2 + 3 + 337 = 342$.

3. Find the value of A such that the following system of equations has an infinite amount of solutions.

   $$9x + 4(y + 3) = 37 + y$$
   $$\frac{4y + 100}{2} - Ax = 8y$$

   Answer: $\boxed{18}$

   Solution: We can simply the first two equations to

   $$9x + 3y = 25$$
   $$Ax + 6y = 50$$

   In order to have an infinite amount of solutions, the two equations should be identical. We note that when we multiply both sides of the first equation by 2, we have $18x+6y = 50$. This means that $A = 18$ will make the two equations have an infinite amount of solutions.

4. Tom R-Vee loves to play volleyball. However, he gets injured a lot. Every time he hits the ball, there is a 20% chance of him getting injured. After he gets injured, he is not able to play for the rest of the day. Find the probability that Tom can not play for the rest of the day after his 4$^{\text{th}}$ shot.

   Answer: $\boxed{\frac{64}{625}}$

   Solution: The probability that Tom doesn't get injured is $1 - 20\% = 80\% = \frac{4}{5}$. Therefore, the probability that Tom doesn't get injured for his first 3 hits but does on his fourth is

$$\frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{1}{5} = \frac{64}{625}.$$

5. How many integers between 1792 and 2022 are multiples of both 6 and 8?

   Answer: $\boxed{10}$

   Solution: The least common multiple of 6 and 8 is 24.

   We note the smallest multiple of 24 between the range we need is $24 \cdot 75 = 1800$. The largest is $24 \cdot 84 = 2016$. Therefore, there are $84 - 75 + 1 = 10$ total multiples of 6 and 8 between 1792 and 2022.

6. The Lu Sports Shop sells different colored tennis balls. Of these, 40% are yellow balls, 25% are blue, 15% are orange balls, 10% are red, and 2022 are green. How many tennis balls are there in total?

   Answer: $\boxed{20220}$

   Solution: Adding the percentages of the non-green balls gets us 40%+25%+15%+10%=90%, which means that the 2022 green balls comprise of the remaining 10% of the total number of tennis balls. Therefore, if the total number of tennis balls is x,

$$\begin{aligned} 0.1 \cdot x &= 2022 \\ x &= \frac{2022}{0.1} \\ &= 20220. \end{aligned}$$

7. The top of a tree is 5 feet taller than the top of a house. The ratio between the two heights is $4 : 5$. How tall is the tree?

   Answer: $\boxed{25}$

   Solution: Since the ratio of 4:5 means that the tree and house differ by one "unit," we know that "unit" is 5 feet because the difference between heights is 5 feet. If each "unit" is 5 feet, then the tree is 5 "units" tall because the tree is taller than the house. The height of the tree is $5 \cdot 5 = 25$ feet.

8. Sean chooses 15 even numbers from 2 to 1000, and Julie chooses 15 odd numbers from 2 to 1000. What is the minimum positive difference between the sum of Sean's numbers and the sum of Julie's numbers?

   Answer: $\boxed{1}$

   Solution: The difference can never be 0, since adding 15 even numbers produces an even number, and adding 15 odd numbers produces an odd number.

   If seven of Sean's even numbers are exactly one less than seven of Julie's odd numbers, and eight of Sean's even numbers are exactly one more than eight of Julie's odd numbers, the positive difference is 1, which is the minimum. For example:

$$\text{Sean: } 2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24, 26, 28, 30, 32$$

$$\text{Julie: } 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31$$

9. A square is circumscribed by a circle. If the area of the square is 64 units^2, what is the area of the circle, rounded to the nearest whole number, in units?

   Answer: $\boxed{50}$

   Solution: Each side length of the square must be $8\sqrt{2}$ units, for a diagonal length of 16, Thus, the circumference is $16\pi \approx 16 \cdot 3.14 = 50.24$. Therefore, the answer is 50.

10. On Eraser island, the flamingo population uses fish and shrimp to feed themselves. The flamingos get their pink color from the shrimp they eat. Currently, there are 33 pink flamingos and 23 gray flamingos. A flamingo eating shrimp will consume 3 a day, while those eating fish will only consume one fish per day. If 11 flamingos eat only shrimp, what is the number of animals eaten over two days?

    Answer: $\boxed{288}$

    Solution: We note that as only 11 flamingos eat only shrimp, there are $33 - 11 = 22$ pink flamingos that eat both shrimp and fish.

    The amount of food eaten in one day is,

$$11 \cdot 3 + 22 \cdot (3 + 1) + 23 \cdot 1 = 33 + 88 + 23 = 144.$$

    Finally, the number of fish they will eat in two days is $144 \cdot 2 = 288$.