



RAMC 2022

Middle School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each.
- This round contains 15 questions to be solved in 45 minutes. Problems towards the end tend to be more difficult than problems toward the beginning.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- All answers must be in a reasonably simplified form.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

1. The Rochester Math Club employs 2022 employees. If its employees form n equal groups, what is the total number of choices for n ?
2. Four ounces of gold can be traded for $23\frac{3}{10}$ pounds of silver. Jake wants to know the monetary value of his $\frac{1}{2}$ pound of gold, when the price of silver is \$20 an ounce. If a pound is 16 ounces, how many dollars is Jake's $\frac{1}{2}$ pound of gold worth?
3. There exists rectangle $ABCD$ such that $\overline{BC} = 2\overline{AB}$. Let E be the midpoint of DC , F be the midpoint of AE , G is the midpoint of BC , and H is the midpoint of BG . Let $[XYZ]$ denote the area of a polygon XYZ . If $[EFHG] = 9$, find \overline{CH} .
4. Five pizzas were ordered for a party. Each pizza is cut into x slices. The party contains 13 people. Each person eats a different, positive number of slices. If each person chooses one of the 5 pizzas to eat from exclusively, what is the smallest possible value of x , assuming that x is an integer?
5. What is the tens digit of 6^{2022} ?
6. Carl picks a random integer from 1 to 3, inclusive. He then picks a random number from 1 to 4, 1 to 5, and so on until 1 to 10. Carl then adds together each number that he picked. Out of all possible combinations, what is the average value of this sum?
7. A new clothing store has opened in Rochester and to celebrate their grand opening, they are offering a discount on jeans and sunglasses. On opening day, 100 people buy jeans and 42 people buy sunglasses. Some people buy both. There are 6 people who buy neither. If one of the people that buys jeans is selected at random, the probability that they also buy glasses is $\frac{3}{10}$. How many people attend the store's opening?
8. A cylinder and a sphere have the same volume. The cylinder has a diameter of 6 feet and a height of 4 feet. What is the diameter of the sphere, in feet?
9. What is the value of $(1 + 2 - 3) + (4 + 5 - 6) + \cdots + (2020 + 2021 - 2022)$?
10. Find the coefficient of x^{18} in the expanded expression for $(x^3 - x^2 + 4x)^5(6x^2 + 5x + 13)^2$.
11. Bob is selling books. He has 20 hardcover books and 16 paperback books. A hardcover book costs \$21, and a paperback book costs \$17. If Bob earned a total of \$360, how many paperback books did he sell?

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12. In isosceles triangle $\triangle ABC$, $\overline{AB} = \overline{BC}$. D is located on line BC such that $\overline{BD} : \overline{DC} = 3 : 2$. M is the midpoint of line AC . Point E is the intersection of lines AD and BM . Let $[XYZ]$ denote the area of a polygon XYZ . If the $[\triangle BAM] = 10$, what is $[\triangle ADC]$?
13. Denise has ten coins. Nine of the coins are fair. The tenth coin is weighted, and has a 75% chance of landing heads. Denise picks one coin at random, and flips it three times. The coin lands heads all three times. What is the probability that the coin she picked was weighted?
14. The six-digit number $\underline{5} \underline{5} \underline{A} \underline{4} \underline{7} \underline{B}$ is divisible by 44. What is the remainder when the number is divided by 9?
15. Triangle $\triangle ABC$ is circumscribed by a circle, with $\overline{AB} = 6$, $\overline{AC} = 8$, and $\angle A = 90^\circ$. Extend the angle bisector from A until it intersects the circle again at point D . What is the length of \overline{CD} ?