RAMC 2022

Middle School Tiebreaker Round

- **SCORING:** The questions in this round are used to break ties, and do not count towards overall scores.
- This round contains 10 questions. Problems towards the end tend to be more difficult than problems toward the beginning.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- All answers must be in a reasonably simplified form.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

1. How many integers between 1024 and 8192 have four distinct digits in strictly decreasing order?

2. Adeline places into a hat one slip of paper labeled with the number "1", two slips of paper labeled "2", three slips of paper labeled "3", and four slips labeled "4". Adeline then randomly takes three slips of paper out of the hat. What is the probability that the product of the numbers on the slips of paper is divisible by 6?

3. Let $b_n = a_n + n - 1$ for all $n \geq 1$. The sequence $b_1, b_2, b_3, \cdots, b_n$ forms an arithmetic series while $a_1, a_2, a_3, \cdots, a_n$ forms a geometric sequence. If $b_{2019} = 2022$, find $a_{2022} - a_1$.

4. Andrew, Damian, Hans, and Michael are counting numbers. Andrew starts with 1, Damian counts the next three numbers, Hans two, then Michael six. After this, the cycle repeats forever, but instead Andrew counts 5 numbers on all his other turns. The game ends when 10,001 is said. How many numbers did Michael say?

5. Michelle has a $\frac{1}{4}$ chance of getting into each of 4 different programs and $\frac{1}{3}$ chance of getting into each of 4 other programs. If getting into each program is independent from another, what are the chances Michelle gets into exactly 1 program?

6. Simplify $\sqrt{5^2 + 4^2 + 3^2 + 2^2 - 1^2 - 1^2 + \sqrt{22^2 + 8^2 + 6^2 + 2^2}}$.

7. Alexander is playing a carnival game. The vendor lets him pick one of 10 identical boxes, two of which have a red ball in them. To win, he must select a box with a red ball. Once he selects a box, the vendor randomly shows him two other boxes without balls in them, and allows him to choose another box or stay with his own. What is the difference in the probabilities of winning using the optimal strategy and the worst strategy?

8. If we arrange ROCHESTER into alphabetical order, we have CEEHORRST, which has no distinct vowels touching each other. We call these words "non-crossed". If we ignore English rules, the letter Y is a consonant, and the letter L can not be in any word, how many non-crossed 3-letter words are there?

9. Richard is a curious guy. When he is interested in a topic, he will run an experiment to learn more about it. When he runs this experiment, there is a $\frac{1}{8}$ chance of leading nowhere and giving up, where he would then give up. There is a $\frac{3}{8}$ chance of having an accident, which he would then do again, but with a $\frac{1}{4}$ chance of leading nowhere and $\frac{3}{4}$ chance of succeeding. Then, there is a $\frac{1}{2}$ chance that he succeeds, in which case he would be intrigued by his discovery and try another experiment. When Richard is interested in a topic, what's the expected number of experiments (repeated experiments count toward the total) he will run?

10. A triangle $\triangle ABC$ is circumscribed by a circle, with $\overline{AB} = 12$, $\overline{AC} = 20$, and $\angle A = 120°$. Let I be the center of the incircle of $\triangle ABC$, and let X be the center of the circle tangent to the circumcircle and BC. Find $\overline{IX}^2$.