RAMC 2025

High School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each, for a total of 20 possible points.
- This round contains 15 questions to be solved in 45 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (E1/E2/MS/HS), and your solution to the problem with explanation.

Do not flip the page until the proctor begins the round!

1. Evaluate $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$.

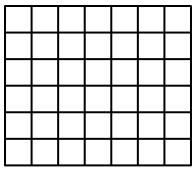
2. Johnny has 5 different red balls and 10 different blue balls. In how many ways can he choose at least one ball such that the number of chosen blue balls is double the number of chosen red balls?

3. Compute
$$\sum_{k=1}^{100} \left(\left\lfloor \frac{3k}{7} \right\rfloor + \left\lfloor \frac{4k}{7} \right\rfloor \right).$$

4. Jimbo shuffles a standard deck of 52 cards fairly, then draws cards off the top until he draws the Ace of Spades. The expected number of aces drawn (including the Ace of Spades) can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$?

5. An integer a is chosen uniformly at random from the set $\{1, 2, 3, 4, 5\}$. Then, b is chosen uniformly from $\{a, a + 1, \cdots, 5\}$. Finally, c is chosen uniformly from $\{b, b + 1, \cdots, 5\}$. What is the expected value of abc?

6. In the 6×7 grid shown below, each cell must be colored either black or white. For how many possible colorings does every 2×2 subgrid contain an even number of each color of cell?



7. Let $P(x)$ be a monic quartic polynomial (i.e. the leading coefficient is 1) such that
$$P(1) = 225, \quad P(2) = 625, \quad P(3) = 1225, \quad P(4) = 2025.$$
Calculate $P(0)$.

8. For $0 \leq x < 2\pi$, the solutions to $\tan^3 x + 4\tan^2 x + 4\tan x + 1 = 0$ sum to $\frac{m\pi}{n}$. Compute $m + n$.

9. Consider a strictly increasing function $f(x)$ whose inputs and outputs are all positive integers. If this function satisfies $f(f(f(x))) = 4x$, what is $f(11)$?

10. Points A, B, and C have coordinates $(0, 0)$, $(4, 4)$, and $(10, 0)$, respectively. Point D satisfies $\angle BDC = 45^\circ$ and has a negative y-coordinate. If ℓ is the maximum possible length of segment AD, find ℓ^2.

11. For some integers a, b, and c, exactly three of the following four statements hold true. Find $a+b+c$.

    (i) A regular polygon with a sides has its interior angles equal to b°.

    (ii) An equilateral triangle with an area of $b\sqrt{3}$ has a side length of c.

    (iii) A right triangle can be formed with side lengths $2a$, c, and 16.

    (iv) A rhombus with one of its angles equal to $9a^\circ$ and a side length of c has an area of b.

12. Determine the unique four digit integer n whose positive divisors $1 = d_1 < d_2 < \cdots < d_{12} = n$ satisfy:

$$3d_3d_9 = d_{12}, \qquad d_1 + d_3 + d_7 = d_3d_4 = d_8.$$

13. Consider a sequence $a_1, a_2, a_3, \cdots, a_k$ with $a_1 = 20$, $a_2 = 25$, and $a_k = 1$. For $3 \leq n \leq k$,

$$a_n = \frac{a_{n-2}a_{n-1} - a_{n-2} - 1}{a_{n-1} - 1}.$$

    Find k.

14. Let S be the set of all possible products of a non-empty subset of $\{2, 3, 5, 7, 11\}$. How many ways are there to select a subset of at least 1 but at most 4 different numbers from S whose product is square?

15. In triangle ABC, let $AB = 52$, $BC = 56$, and $CA = 60$. Let H be the orthocenter and O be the circumcenter of $\triangle ABC$. Let M be the midpoint of BC. Compute the area of quadrilateral $AOMH$.