Rochester Math Club Placement Test (Novice)

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1 Introduction

This is a practice test for your personal use to determine which math circle you decide to enroll in. If you choose a level that you feel is ill-suited, please tell us, and we will make arrangements to change the level.

Set aside 90 minutes to attempt these 30 problems. Problems range from easy to difficult, although the exact order is not set (question 24 could be easier than question 17). Please **DO NOT USE** a calculator! Good luck!

After you finish, please email us your answers, and solutions will be available to you once you become a member. Your score will not be used to determine your level placement and will be kept private.

2 Problems

Instructions: The word "exact" in the questions call for an answer in simplest form. For example, instead of answering 11.8211, write instead $5 + \sqrt{5} + \sqrt[3]{5} + \sqrt[4]{5} + \sqrt[5]{5}$. Good luck.

1. What is $1 + 2 + 3 \cdots + 8 + 9 + 10$?

2. In how many ways can 57 be written as the sum of two prime numbers?

3. A rectangle has a width of 3. If the length is three times the width, calculate the perimeter of the rectangle.

4. 4 people share 24 slices of pizza. If person A eats the same number of slices as persons B, C, and D combined, and if $B = C = D$, how many slices did person B eat?

5. Find the value of x if $4x + 28 = 12$.

6. Different shapes represent different positive integers. Find the sum of the square and the triangle.

7. A triangle has a base of 6. If the height is half of the base, what is the area of the triangle?

8. How many positive integers less than or equal to 60 are multiples of 3 or 4?

9. How many different ways can 16 be written as a sum of 4 different positive integers?

10. A square and a regular hexagon have the same perimeter. What is

the ratio of the area of the hexagon to the area of the square?

11.   Billy can finish a job in 3 hours. Cristiano can finish the same job in 2 hours. How much time, in minutes, would it take them to finish the job if they worked together?

12.   Among all the kangaroos, the lightest two, who have the same weight, share $\frac{1}{4}$ of the total weight of the kangaroos. The heaviest three, who have the same weight, share $\frac{3}{5}$ of the total weight. How many kangaroos are there in total? (written by Tony Liu)

13.   A circle is inscribed in a $3 - 4 - 5$ triangle. Find the area of the circle, expressed in terms of π.

14.   27 unit cubes are arranged into a $3 \times 3 \times 3$ cube. All the faces of the larger cube are painted blue. When randomly choosing a small cube, what is the probability of selecting one with two faces painted?

15.   The sum of the ages of Bob's 4 kids is K. K is also equal to Bob's age. X years later, the sum of the kids' ages is 3 times Bob's age. What is $\frac{X}{K}$?

16.   There is a total of 25 humans and dogs in a neighborhood. Assume every human has 2 legs, and every dog has 4. If there are 60 legs in total, what is the positive difference between the numbers of humans and dogs?

17.   The number $\underline{A}\ \underline{3}\ \underline{7}\ \underline{6}\ \underline{B}\ \underline{0}$ is divisible by 99. Find all possible ordered pairs (A, B).

18.   Let the mixed fraction $A\frac{B}{C}$ represent the value of $\frac{2+4+6+....4036}{1+3+5+....2017}$. Find the sum $A + B + C$.

19.   The angles in a triangle have the ratio $2 : 3 : 4$. What is the angle measure of the largest angle?

20.   A group of teenagers and adults are discussing how to take over the world. Initially, 25% of the group are teenagers. After 3 adults and 2 teenagers leave, 20% of the group are teenagers. How many adults were there initially?

21.   Four integers a, b, c, d, not necessarily different, are chosen randomly between 1 and 36, inclusive. What is the probability that $ab - cd$ is odd?

22.   In the addition equation below, different letters represent distinct non-

negative digits (1-9). Given that $D = 1$, find the four-digit number $\underline{MATH}$[1]

$$
\begin{array}{r}
H\,A\,R\,D \\
+\ G\,O\,O\,D \\
\hline
M\,A\,T\,H
\end{array}
$$

23.   A number is called a "good number" if all of its factors excluding it-self add up to the number itself. For example, 6 is a "good number" since $6 = 1 + 2 + 3$. Find the next "good number" after 6.

24.   In triangle XYZ, the height from X is drawn, hitting segment YZ at a point O. Then, a height of triangle XOZ from O is drawn, hitting side XZ at a point P. Given that $XY = 10$, $OY = 6$, $XZ = 17$, find exactly the length of OP.

25.   Determine exactly $2x_1 + x_3$ if x_1, x_2, x_3, x_4, and x_5 satisfy the system of equations below.

$$3x_1 + x_2 + x_3 + x_4 = 8$$

$$x_1 + 3x_2 + x_3 + x_4 = 21$$

$$x_1 + x_2 + 3x_3 + x_4 = 36$$

$$x_1 + x_2 + x_3 + 3x_4 = 54$$

26.   Lebron swims and runs at constant rates. He runs three times as fast as he swims. In a race, Lebron completes the swimming part in 30 minutes and the running part in 2 hours. Find exactly the ratio of the distance he travels swimming to the distance he travels running.

27.   A certain unfair six-sided die with faces numbered $1 - 6$ has the property that the probability of rolling a number is proportional to the number's value. When two such dies are rolled, what is the probability that the sum of the two numbers is 6?

28.   When a number is divided by 3, the remainder is 2. When the same number is divided by 5, the remainder is 4. When the same number is divided

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[1] There are multiple solutions; any that work are acceptable.

by 7, the remainder is 1. Find the second least positive number that satisfies these conditions.

29. Richard has four identical fair coins with a distinguishable head and tail. What is the probability that at least two of the coins land heads?

30. A right regular octagonal pyramid $ABCDEFGHI$ with A as the vertex has a base with length 4. Given that $AB = 10$, find exactly the volume of the pyramid.

This is the end of the test. Please check go back and check your answers until your 90 minutes are up.