RAMC 2022

Elementary I Tiebreaker Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

Executive Editors

Arden Peng Michael Huang Golden Peng

Problem Writers

Albert Hu	Arden Peng	Felix Lu	Kevin Yang
Alexander Voskoboev	Aurora Wang	Golden Peng	Matthew Chen
Ana Milosevic	Ben Weingarten	Hans Xu	Michael Huang
Andrew Sun	Christine Song	Jason Ding	Michelle Cao
Andrew Yan	Ethan Zhang	Katherine Zhu	William Wang

Graciously Reviewed By:

Frank Lu Lei Zhu Leo Xu

Liya Huang Nan Feng

1. Kevin is filling up a pool. The pool is 20 feet long, 10 feet wide, and and varies in depth. Inside the pool, there's a shallow zone 10 feet long, 10 feet wide, and 5 feet deep. The remaining part has a depth of 7 feet. How many cubic feet of water does Kevin need to fill up the pool.

   Answer: $\boxed{1200}$

   Solution: We can think of the pool as two pools: the shallow zone, and the not shallow zone. The shallow zone contains $10 \cdot 10 \cdot 5 = 500$ cubic feet of water, and the not shallow zone contains $(20 - 10) \cdot 10 \cdot 7 = 700$ cubic feet of water, for a total of 1200 cubic feet.

2. Bob is sharing a pizza with his friends. He takes one third of the pizza, and gives the remainder to Claire. She takes a third of it, and gives the remaining part to Jenny, who eats half of it. What part of the original pizza does Jenny have left?

   Answer: $\boxed{\frac{2}{9}}$

   Solution: Bob takes $\frac{1}{3}$ of the pizza, giving $\frac{2}{3}$ to Claire. Claire takes a third of that, which is $\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$ of the original pizza. Claire passes on $\frac{2}{3} - \frac{2}{9} = \frac{4}{9}$ of the pizza to Jenny. She takes half of it, which is $\frac{2}{9}$ of the original pizza, leaving $\frac{2}{9}$ of the pizza remaining.

3. John has a bag of 10 coins. Six are quarters, two are dimes, one is a nickel, and one is a penny. Find the probability that John draws a dime.

   Answer: $\boxed{\frac{1}{5}}$

   Solution: There are 2 dimes and 10 total coins, which means that the probability of drawing a dime is $\frac{2}{10} = \frac{1}{5}$.

4. Ben uses the digits 0, 1, 3, 5, 7, and 9 to create two 3 digit numbers. What is the greatest possible difference between the two numbers, if no digit can be used more than once?

   Answer: $\boxed{872}$

   Solution: We want the small number to be as small as possible and the big number to be as big as possible. For the small number, we use 1 as the hundreds place, 0 as the tens place, and 3 as the ones place, giving us 103. For the big number, we use 9 as the hundreds place, 7 as the tens place, and 5 as the ones place, giving us 975. Therefore, the greatest difference is $975 - 103 = 872$.

5. If Kelly bought a shirt for 12 dollars when the shirt was 25% off, what was the original price of the shirt?

   Answer: $\boxed{16}$

   Solution: Since Kelly bought the shirt for 12 dollars when it was 25% off, that means that 12 is 75% of the full price, or $\frac{3}{4}$ of the full price. That means that the full price would be $12 \cdot \frac{4}{3} = 16$.

6. Forrest indulges in a box of chocolates for 5 days. The box includes 20 Cocoa Truffles, 20 Chocolate Caramels, and 10 Dark Chocolate pieces. Each day, he eats either 3 Cocoa Truffles, 2 Chocolate Caramels, or 1 Dark Chocolate piece. At the end of the 5 days, he gives the remaining 36 pieces to a friend. When Forrest receives the box, there are t Cocoa Truffles, c Chocolate Caramels, and d dark chocolates left. Find $tc + d$.

   Answer: $\boxed{410}$

   Solution: As we are talking about the state of the original box, this means that $t = 20$, $c = 20$, and $d = 10$. Therefore, $tc + d = 20 \cdot 20 + 10 = 410$.

7. A 3 by 3 cube is made of twenty-seven $1 \times 1 \times 1$ cm cubes. If all 4 of the corner cubes are removed, what is the surface area of the cube?

   Answer: $\boxed{54}$

   Solution: The area remains unchanged. This is because each corner cube has 3 faces exposed, which are the only faces that count towards surface area. If we remove the corners, we remove these faces, but we also expose three more faces on other smaller cubes. Thus, the surface area of the cube remains the same at $6 \cdot 3 \cdot 3 = 54$ square inches.

8. The lines $x^2 + 9 = y$ and $y - 2x = 17$ intersect at two points. Let the intersection point in the 2$^{\text{nd}}$ quadrant be $(-a, b)$. Find $a + b$.

   Answer: $\boxed{15}$

   Solution: We can substitute $y = x^2 + 9$ into the second equation to get

$$\begin{aligned} x^2 - 2x + 9 &= 17 \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0. \end{aligned}$$

   So $x = 4$ or -2. However, as we need to find the intersection in the second quadrant, we go with $x = -2$ and $a = 2$. Therefore, $y = x^2 + 9 = 4 + 9 = 13$. So $a + b = 2 + 13 = 15$.

9. A 40-liter paint mixture is made, with 30% being red, 15% being yellow, the remaining is blue. What percent of the mixture is red paint, if 10 liters of red paint are added to the mix.

   Answer: $\boxed{44}$

   Solution: We know that there are $30\% \cdot 40 = 12$ liters of red paint in the mix. Since 10 more liters of red paint was added, there are $12 + 10 = 22$ liters of red paint in the new mixture. As the total amount of paint increases from 40 to 50, the percentage of red paint in the new mix is $\frac{22}{50} = 44\%$.

10. In a math contest, each student is required to answer 20 problems. For every question answered correctly, the student gets 2 points. If the student answers incorrectly, then the student loses 1 point. Jack answered all 20 questions and earned a score of 22 points. How many questions did Jack get correct?

    Answer: $\boxed{14}$

    Solution: Let the number of correct answered questions be a, and the number of incorrect answered questions be b. We can write

$$a + b = 20$$
$$2a - b = 22.$$

Adding both equations, we have $3a = 42$, which means that the number of questions Jack got correct is $\frac{42}{3} = \boxed{14}$.