RAMC 2022

Elementary II Individual Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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1. Elise is playing on her phone. Its battery charge is currently at 50, out of a maximum of 100. While the phone is charging, its battery charge increases by 2 every minute, but since Elise is playing, its charge goes down by 1 every 2 minutes. How long does it take for Elise's phone to fully charge? Express your answer in the form minutes:seconds.

   Answer: $\boxed{\text{33:20 or 33m20s}}$

   Solution: As Elise's battery is at 50, it needs to go up by 50 in order to be fully charged. We can calculate the amount of battery every minute.

$$\frac{2}{1\text{ min}} - \frac{1}{2\text{ min}} = \frac{4}{2\text{ min}} - \frac{1}{2\text{ min}} = \frac{3}{2\text{ min}}.$$

   If it needs to go up by 50, we have $\frac{50}{x} = \frac{3}{2\text{ min}}$, so $x = 33\frac{1}{3}$ minutes. As an minute is 60 seconds, this gives us 33 minutes and $\frac{1}{3} \cdot 60 = 20$ seconds.

2. A target-shaped figure has 3 concentric circles, as shown in the diagram. The inner and outer-most circles are colored red, while the smaller inside ring is colored gray, like shown in the diagram below. The distance between each circle is 5 units. If the radius of the inner circle is also 5 units, find the area of the red sections, in units^2.

   Answer: $\boxed{150\pi}$

   Solution: As the only non-red section of the diagram, we can calculate the area of the blue section and then subtract that from the total. We notice the radius of the whole circle is $5+5+5 = 15$, which gives us an area of $15^2\pi = 225\pi$.

   To calculate the area of the gray section, we notice that the radius is $5+5 = 10$, and the radius of the innermost circle is 5. This makes the area of the gray section to be $10^2\pi - 5^2\pi = 100\pi - 25\pi = 75\pi$.

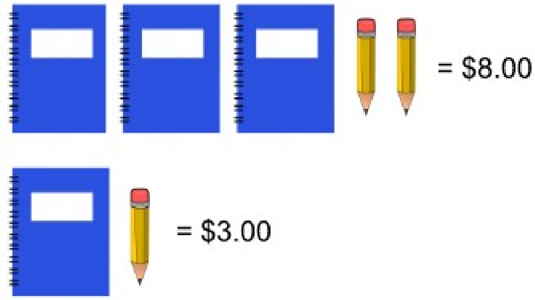
   Therefore, the total area of the red section is $225\pi - 75\pi = 150\pi$.

3. Brady went to the local grocery store and bought a bag of colored candies. Inside of the bag, he finds 15 blue, 12 red, 6 green, and 12 purple candies. He blindly grabs 2 out of the bag, one after the other, without replacement. Find the probability that both the candies Brady grabbed were blue.

   Answer: $\boxed{\frac{7}{66}}$

   Solution: The probability of the first candy being blue is $\frac{15}{15+12+6+12} = \frac{15}{45} = \frac{1}{3}$. The second candy can't be the first candy, so there are 14 blue candies and 44 total candies remaining. The probability that the second one is also blue is $\frac{14}{44} = \frac{7}{22}$. Therefore, the total probability is the product of the 2, which is $\frac{1}{3} \cdot \frac{7}{22} = \frac{7}{66}$.

4. Use the diagram below to find the cost of one notebook, in dollars.



Answer: $\boxed{\$2}$

Solution: As one notebook and one pencil is \$3, so two notebooks and two pencils are worth $2\cdot\$3=\$6$. As three notebooks and two pencils are worth \$8, subtracting $\$8-\$6$ gives us the value of just one notebook, which is \$2.

5. Evaluate the expression $2022 \div ((3 \times 0 + 6 - 2) \times 7 + 2 - 7 \times 3 - 6)$.

Answer: $\boxed{674}$

Solution: Using PEMDAS,

$$\begin{aligned}
2022 \div ((3 \times 0 + 6 - 2) \times 7 + 2 - 7 \times 3 - 6) &= 2022 \div ((0 + 6 - 2) \times 7 + 2 - 7 \times 3 - 6) \\
&= 2022 \div ((4) \times 7 + 2 - 7 \times 3 - 6) \\
&= 2022 \div (28 + 2 - 21 - 6) \\
&= 2022 \div (3) \\
&= 674.
\end{aligned}$$

6. Susie is swimming a 2 kilometer race, where 1 kilometer = 1000 meters. She swims the first 200 meters in 3 minutes. For every 200 meters following the initial one, her pace continuously slows as she has to swim 2 more seconds than the previous 200. After completing 1 kilometer, Susie's goggles fall off, and she has to stop for 7 seconds to fix them. How long does it take for Susie to swim the whole 2 kilometers? Express your answer in seconds.

   Answer: $\boxed{1897}$

   Solution: We note that 2 km $= 2 \cdot 1000$ m $= 2000$ m, which creates ten 200 meter intervals.

   Susie swims the first 200 meters takes 3 minutes $= 180$ seconds. Each one will add two seconds, so we have,

$$180 + (180 + 2) + (180 + 4) + \cdots + (180 + 18) = 180 \cdot 10 + 2 + 4 + 6 + \cdots + 18 = 1800 + 90 = 1890.$$

   Finally, as Susie's goggles fall off, adding an additional 7 seconds. Therefore, her total time is $1890+7 = 1897$ seconds.

7. Timothy and Colin love dice games. They design a game where each person rolls 2 dice and whoever has the higher sum of the 2 numbers shown on the dice wins. Timothy rolls a 3 and 4. Find the probability that Colin wins over Timothy.

   Answer: $\boxed{\frac{5}{12}}$

   Solution: We write out the table for the possible sums Colin can get.

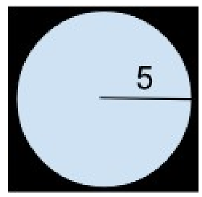
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

   Since there are 15 numbers higher than Timothy's sum of $3 + 4 = 7$, the total probability is $\frac{15}{36} = \frac{5}{12}$.

8. John wants to draw the smallest square around a circle of radius length 5, such that the circle is completely within the square. What is the area of the square that he should draw?

   Answer: $\boxed{100}$

   Solution: The diagram below is the smallest square we can draw, as it touches all 4 sides. Since the diameter is $5 \cdot 2 = 10$, and the diameter is equal in length to a side of a square, the answer is $10 \cdot 10 = 100$.



9. Christine has a normal pack of 54 cards, but the sizes of each card varies. Like a normal deck, there are 2 jokers, 12 face cards (including 4 jacks, 4 queens, and 4 kings), and the rest are numbered cards. A numbered card in this deck is a 4 units by 5 units rectangle. A face card is in a triangular shape with a base of 3 units and height 8. Finally, a joker is a square with a side length of 15 units. If all the cards are laid out on an infinite table, what is the maximum area the cards can cover? Express your answer in units^2.

   Answer: $\boxed{1394 \text{ units}^2}$

   Solution: We need to calculate the total area of all the cards. The two jokers make an area of $2 \cdot 15 \cdot 15 = 450$. Each face card has an area of $3 \cdot \frac{8}{2} = 12$, so they have a total of $12 \cdot 12 = 144$. There are $54 - 2 - 12 = 40$ numbered cards, each with an area of $4 \cdot 5 = 20$, for a total area of $40 \cdot 20 = 800$. As long as the cards don't overlap, the maximal area is reached, which is $450 + 144 + 800 = 1394$ units^2.

10. Leo is trying to improve his basketball skills by continuously shooting hoops. At the start, he can only hit one after missing 39 in a row. However, after each set of 120 shots, he can make a basket in 10 less tries. For example, after 120 shots, he can make the basket on his 30$^{\text{th}}$ shot, and after 240 shots, he can make the bucket on the 20$^{\text{th}}$. After exactly 450 shots, how many free throws has he made?

    Answer: $\boxed{22}$

    Solution: In his first 120 shots, Leo makes $\frac{120}{40} = 3$ shots. For his 121$^{\text{st}}$ to 240$^{\text{th}}$ shots, he makes $\frac{120}{30} = 4$ more shots. For the next 120, he makes $\frac{120}{20} = 6$ shots. From the 361$^{\text{st}}$ to the 450$^{\text{th}}$ shot, he makes $\frac{90}{10} = 9$. In total, he successfully lands $3 + 4 + 6 + 9 = 22$ total shots.

11. Given that a mile is equal to 5280 feet, how many inches are in $\frac{1}{8}$ of a mile?

    Answer: $\boxed{7920}$

    Solution: Firstly, $\frac{1}{8} \cdot 5280 = 660$, which is the number of feet in $\frac{1}{8}$ of a mile. We need to convert feet to inches with the conversion 1 feet = 12 inches. Therefore, we have $660 \cdot 12 = 7920$.

12. A snail is climbing up a pipe that is 11 meters long. Each day, the snail can climb up 3 meters during the day, and slides down 1 meter during the night when the snail is sleeping. How many days does it take for the snail to reach the top the pipe?

    Answer: $\boxed{5}$

    Solution: The snail climbs at a rate of 3 meters per day, then falls down by 1. Making a table of the snail's actions, we can see the snail climbs to 11 meters on day 5.

Day #	1	2	3	4	5
Day	3	5	7	9	11
Night	2	4	6	8	10

13. Japanese Beetles are an invasive species. With every passing day in a forest, the beetles will increase to 3 times the current population. However, if the beetles exceed a population of 150 on any given day, the next day, half the beetles will die, and they will not reproduce until the day after that. If there are currently 64 beetles in the field, how many beetles will be in the field after a week passes?

    Answer: $\boxed{108}$

    Solution: We can make this chart to reflect the problem.

Day	0	1	2	3	4	5	6	7
Count	64	192	96	288	144	432	216	108

Therefore, there are 108 beetles a week from the initial day.

14. Two painting companies are hired to paint lines on a 7 mile road. Company A paints the road at the speed of 528 inches per minute. Company B paints the road at 352 yards per hour. The two companies start painting at opposite sides of the road. Using the conversions 3 feet = 1 yard and 5280 feet = 1 mile, how many minutes does it take them to complete the job?

Answer: $\boxed{600}$

Solution:

Every hour, the two companies will complete $580 \cdot 60 \text{ in} + 352 \text{ yd} = 880 \text{ yd} + 352 \text{ yd} = 1232 \text{ yd}$, A mile is 1760 yards, so the amount of time it'll take them to complete 7 miles is:

$$\begin{aligned}\frac{1760 \text{ yd}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{1232 \text{ yd}} \cdot 7 \text{ mi} &= \frac{12320}{1232} \text{ hr} \\ &= 10 \text{ hr} \\ &= 600 \text{ min.}\end{aligned}$$

15. Forrest indulges in a box of chocolates for 5 days. The box includes 20 Cocoa Truffles, 20 Chocolate Caramels, and 10 Dark Chocolate pieces. Each day, he eats either 3 Cocoa Truffles, 2 Chocolate Caramels, or 1 Dark Chocolate piece. At the end of the 5 days, he gives the remaining 36 pieces to a friend. When Forrest gives the leftovers to his friend, there are t Cocoa Truffles, c Chocolate Caramels, and d Dark Chocolates left. Find $t + cd$.

Answer: $\boxed{188}$

Solution: Forrest eats $20 + 20 + 10 - 36 = 14$ total chocolates over the 5 days. We should note that if he ate Cocoa Truffles every day, he would only consume $3 \cdot 5 = 15$. This means that the only possible combination of chocolates eaten are 4 days of Cocoa Truffles and 1 day of Chocolate Caramels.

We can calculate t, c, and d now. We see that $t = 20-4\cdot3 = 8$, $c = 20-1\cdot2 = 18$, and $d = 10-0\cdot1 = 10$. Therefore, $t + cd = 8 + 18 \cdot 10 = 188$.