



# RAMC 2022

## High School Team Round

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- **SCORING:** The first 5 questions are worth 2 points each, and last 5 questions are worth 3 points each.
- This round contains 10 questions to be solved in 25 minutes. Problems towards the end tend to be more difficult than problems toward the beginning.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- All answers must be in a reasonably simplified form.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

1. Cathy writes on the board all multiples of either 24 or 42, or both, on a whiteboard between 1 and 1000. What is the sum of the numbers on the board?
2. Let  $n = 2022^{22}$ . Find the number of positive divisors of  $n^2$  that are less than  $n$  but are not divisors of  $n$ .
3. There is a sequence of positive numbers  $a_1, a_2, a_3, \dots, a_m$  such that  $a_1 = \frac{233}{9}$ ,  $a_2 = 18$ ,  $a_3 = 16$ , and  $a_m = 0$ , which follows the relation

$$a_{n+3} = a_{n+1} \left( \frac{a_n}{a_{n+2}} - 1 \right) \text{ for } n = 1, 2, 3, \dots, m.$$

Find the value of  $m$ .

4. Four spheres of equal radius are constructed in a cone such that they are all tangent to the base of the cone, the lateral surface of the cone, and to two other spheres. A fifth sphere with twice the radius is stacked on top of the cones such that it is tangent to all 4 spheres and to the lateral surface of the cone. What is the ratio of the height of the cone to the radius of the small spheres?
5. Balrog the frog is at the bottom of a staircase. He needs to ascend a total of 13 steps (the length of the staircase) to get to the top. His jumping power is amazing, so he can traverse any number of steps in a single jump. However, his parents get mad at him for taking the stairs too quickly if it takes less than 4 jumps to reach the top of the staircase. In how many different ways can Balrog reach the top of the staircase without his parents getting mad at him?
6. Find the largest prime factor of

$$\sum_{i=3}^{50} \left[ 6 \binom{i}{3} + (i-1) \right]$$

7. A regular dodecagon (12-sided polygon) is inscribed within a circle. To the nearest integer, what percent of the area of the circle is outside of the dodecagon?
8. Randy the Ant walks along a coordinate plane from  $(0, 0)$  to  $(5, 5)$ . Each step takes him one direction along the lattice. How many ways are there for Randy to get to  $(5, 5)$  in 12 steps?

9. Each of the following boxes in the grid below are filled with a digit from 1 to 9. Like a crossword, the number that corresponds to a given clue is the 4 digit number in the direction given by the clue starting at the grid box with the same small number as the clue's. What is the product of the upper left and the lower right digits of the grid below?

1	2	3	4
5			
6			
7			

**Across:**

1-Across: Prime

5-Across: Factor of 10753344

6-Across: Digits sum to 29

7-Across: Cube

**Down:**

1-Down: Multiple of 9

2-Down: Multiple of 241

3-Down: Multiple of 24

4-Down: Multiple of 44 between 4000 and 8000

10. The polynomial  $P(x) = x^{2022} + x^{2021} - 3x + 1$  has complex roots  $r_1, r_2, \dots, r_{2022}$ . Find the value of  $(r_1^2 + 1)(r_2^2 + 1) \cdots (r_{2022}^2 + 1)$ .