RAMC 2023

High School Tiebreaker Round

- **SCORING:** The questions in this round are used to break ties, and are not necessarily weighted the same.
- This round contains 10 questions to be solved in 30 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

1. Perry's family has 5 people in total. In a room, there are 11 red socks, 9 yellow socks, 7 orange socks, 5 green socks, 3 blue socks, and 1 purple sock. Each can be worn on either foot. What is the minimum number of socks that Perry must draw such that it is guaranteed that the family can all wear the same combination of not-necessarily matching socks?

2. The roots of the equation $x^5 + 72x^4 + Ax^3 + Bx^3 + Cx^2 + Dx + E = 0$ form a geometric progression. The sum of their reciprocals is 8. Find the value of $|E|$.

3. Circles P and Q have diameters AB and CD, respectively, and they share a common tangent AD. Circle M is constructed, concentric to circle P, to pass through point D. Circle N is constructed, concentric to circle Q, to pass through point A. Given that $AB = 4CD$, the positive difference in area between circles M and N can be expressed as an integer multiple, k, of the area of circle Q. Find the value of k.

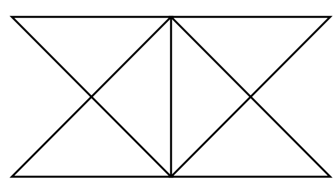
4. Consider trapezoid $ABCD$ with $AB \parallel CD$ and $\angle BAD = \angle ADC = 90°$. Let $AB = 3$, $CD = 6$, and $AD = 8$, and consider a point E on AD such that $BE + CE$ is minimized. If $\sqrt{k} = BE + CE$, find k.

5. There is a function $f(x)$ that is defined for all non-negative integers x. If $f(x+y) = f(x)+f(y)-2f(xy)$, and $f(1) = -1$, then what is the value of $f(162)$?

6. Compute ab, if a and b are positive integers satisfying:

$$\gcd(a^3, b^3) + \gcd(a, b) - 2 = 0,$$
$$\text{lcm}(a^2, b) + \text{lcm}(a, b^3) - 228 = 0.$$

7. A triangular bipyramid is folded from the net shown, where each face is an isosceles right triangle with unit-length legs. The radius of the largest sphere that can fit inside can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.



8. For how many 4-digit integers does the sum of two of its digits equal the sum of the other two digits?

9. In a 30-60-90 right triangle, the length of the hypotenuse is 4. Let d be the distance from the vertex of the right angle to the point of intersection of the angle bisectors. The value d^2 can be expressed as $a + b\sqrt{c}$ where a, b, and c are integers and c has no square factors other than 1. Find $a + b + c$.

10. A sequence of real numbers with $a_0 = 1$ and $a_1 = 5$ is defined by the recursion $a_n = \dfrac{a_{n-1}^2}{a_{n-2}} + 5a_{n-1}$. Find the number of tailing 0s that a_{12} has.