2nd Rochester Area Math Competition 2020

12 September 2020

Middle School Individual

1. Harry writes 10, 11, 100, 110, and 1000 on the class whiteboard. Harry then erases two of the numbers and heads back to his seat. What is the largest possible sum of the remaining numbers on the board?

2. In a racing game, players can choose between a kart with 4 green wheels and a bike with 2 red wheels. On the game map of Enchanted Valley, there are a total of 10 vehicles and 26 wheels. How many bicycles are there in Enchanted Valley?

3. Video games initially occupy 5% of the hard drive space on Felix's computer. After downloading a new computer game *Fort Day*, which adds 18 gigabytes, video games now take up 15% of the hard drive space. How many gigabytes can be stored inside his computer's hard drive in total?

4. Jenny has two music playlists on Spotify: *Summer Music* and *Winter Music*. No two songs are shared between the two playlists. The *Summer Music* playlist has 15 happy songs and 5 sad songs, while the other playlist has 10 happy songs and 10 sad songs. She combines both playlists and selects a random song from the available 40 songs. Given that the song was sad, the probability that the song came from the *Summer Music* playlist came be expressed in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute $p + q$.

5. Ethan is playing on a football field that is rectangular shaped with a length of 120 m and width of 80 m. He travels diagonally across the field in 20 seconds, jogging from one corner of the field to the opposite corner in a straight line. He then walks back to the original corner, which takes 40 seconds. His average speed in meters per second for the round trip can be expressed in the form $\frac{a}{b}\sqrt{c}$, where a and b are relatively prime positive integers and c has no factors that are squares of a prime. Compute $a + b + c$.

6. For a real number x, the positive side lengths of an isosceles triangle are $22 - x^2$, $-x^2 + 6x + 10$, and $-x^2 + 12x - 14$. What is the largest possible perimeter of the triangle?

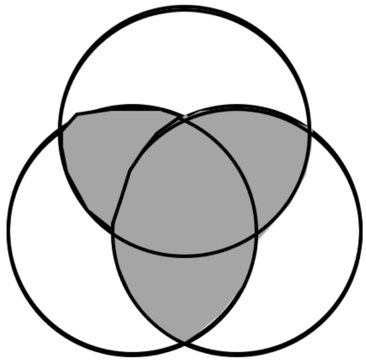
7. 10 students are on a field trip to the Grand Canyon. They divide into two group of 5, with one group heading to Havasu Falls and another group heading to the Rim Trail. Suppose that two of the students, Chris and Tiffany, insist on being in the same group, while a different pair of students, John and Cynthia, insist on being in different groups. How many distinct ways are there to assign the students to locations?

8. A function g satisfies $4g(x) - \frac{1}{x}g(-\frac{1}{x}) = 17$ for all real values of x not equal to 0. The value of $g(3)$ can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

9. A cube has vertices placed at $(2,3,4)$ and $(3,3,4)$. The product of all possible positive volumes of the cube can be expressed in the form $\frac{a}{b}\sqrt{c}$, where a, b, and c are positive integers; a and b are relatively prime; and c is not divisible by the square of any prime. Compute $a+b+c$.

10. A magician is standing at $(0,0)$ in the xy coordinate plane. From any point (x,y), she can use a spell to travel to $(x,y+1)$ or to travel to $(x+1,y)$. Suppose that she journeys from $(0,0)$ to $(4,5)$ using a sequence of spells. If she selects one of these sequences at random, the probability that she will pass by a crater at $(2,3)$ can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Compute $a+b$.

11. Three congruent circles, each with radius 2 feet, are placed as shown such that each circle intersects with the centers of the other two circles. The area of the shaded region in feet squared, which is the area shared between at least two of the circles, can be expressed as $m\pi+n\sqrt{q}$, where m, n, and q are integers and q is not divisible by the square of any prime. Compute $m^2+n^2+q^2$.



12. Two trains, one red and the other turquoise, are heading in opposite directions along parallel tracks near a mountain's summit. The red train is traveling at 50 feet per second while the green train is traveling at 100 feet per second. Starting from the time that the fronts of each train pull even, it takes a third of a minute for the backs of the train to pull even. If the red train is four times longer than the turquoise train, what is the length in feet of the red train?

13. Find the smallest integer $n>1$ such that n^3 has more than 64 divisors.

14. Frank is enrolled in an university physics course. On the first day, he waves to the class. From each day forward, he will either wave to the class or not. Each day, he will wave with $\frac{3}{4}$ probability if he waved the previous day. If he did not wave the previous day, he will wave with $\frac{1}{4}$ probability. The probability that he will wave on the 8th day can be expressed as a simplified fraction $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute $p+q$.

15. The vertices of a triangle lie on the graph of $f(x)=3^x$ and their x-coordinates form an arithmetic sequence of positive integers. Suppose that the area of the triangle is 1728. Compute the x-coordinate of the rightmost vertex.