RAMC 2023

Middle School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each, for a total of 20 possible points.
- This round contains 15 questions to be solved in 45 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

1. Let $x + \frac{1}{x} = 4$. What is the value of $x^4 + \frac{1}{x^4}$?

2. Jerry mixes a 20-ounce drink, originally comprised of 44% eggnog, 20% water, 10% lemonade, and 26% milk, by weight. If he adds 5 more ounces of water to his drink, what percent of the new drink is water?

3. Owen needs to purchase exactly 24 apples. The store only sells apples in packs of 1, 5, or 10. The 1-pack costs \$1.35, the 5-pack costs \$6.25, and the 10-pack costs \$13.30. How many packs of apples should Owen purchase if he wants to spend the least amount of money?

4. A cat has 8 bowls of water, arranged in a line. While at home alone, it caught 4 distinct mice. Before the owners get home, the cat wants to place each mouse into its own bowl of water. How many ways can the cat place the four mice if it doesn't place mice in adjacent bowls?

5. How many positive integer factors does the number 1260 have?

6. The Rochester Math Festival is hosted on the second Saturday in September of every year. RMF 2099 will occur on the date $9/X/99$. Find X.

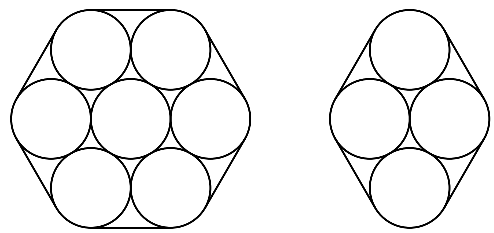
7. Alex is taking a 5-question true-false math test and decides to randomly guess. He has a 50% chance of guessing any particular question correctly. The probability that Alex gets at least 4 questions right can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.

8. If Andrew can eat a plate of pasta every 6 minutes and Willy can eat a plate of pasta in 3 minutes, how many minutes would it take Andrew and Willy to eat one plate of pasta together?

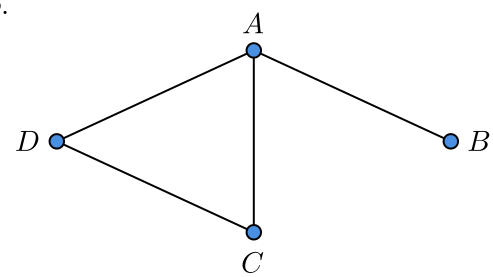
9. Mr. Anderson must screen 100 students for an interview. If each screening takes exactly 90 seconds, and the time on a digital clock is 10:00AM when he begins screening the first student, what is the sum of the digits on the digital clock when he is finished with the last student?

10. How many non-similar convex nonagons (9-sided polygons) have internal angles whose degree measures are distinct positive integers that form an arithmetic sequence?

11. Seven hockey pucks with radius 1 are initially held together by a tight rubber band, as shown on the left. The rubber band is so tight that three pucks pop out of formation, and the remaining pucks snap into the shape shown on the right. The difference in area enclosed by the rubber band in these two configurations can be expressed as $a + b\sqrt{c}$, where a, b, and c are integers and c has no square factors other than 1. Find $a + b + c$.



12. An ant starts on node A in the graph below. Every second, the ant will move to one of the connected nodes, choosing one uniformly at random if there are multiple options. For example, if the ant is at node C, it will move to node A with a $\frac{1}{2}$ chance and it will move to node D with a $\frac{1}{2}$ chance. The probability that the ant is at node A after three seconds can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.



13. At Rochester's (future) ZipRail station, high speed trains arrive every hour, from both the north and the south. Each train stops for 5 minutes to let passengers deboard and board. Michael chooses a random time to arrive at the station. The probability that both trains are at the station can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.

14. Matthew and Dan flip a fair coin repeatedly heads or tails until one of them wins. Matthew wins once the coin lands two consecutive heads. Dan wins if the coin lands three consecutive tails. The first flip was heads. If the probability that Dan wins can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.

15. The real solutions to the equation $4x^4 + 4 = 31x^3 + 31x$ are of the form $a \pm \sqrt{b}$. Find $a + b$.