2nd Rochester Area Math Competition 2020

12 September 2020

Middle School Team

1. Meng uploads a dance video to a social media platform. On the day he posted, the ratio of likes to comments was $4:1$. The ratio of likes to comments is now $10:3$, with 200 likes and 300 comments added since the day he posted. How many likes does Meng's video currently have?

2. An artist experiments with a triangle-shaped canvas that has side lengths 30 inches, 40 inches, and 50 inches. She draws the largest circle she can fit inside this canvas, painting it so that it resembles the moon. What is the radius of this circle in inches?

3. William is a player on his high school football team, the Tigers. In the recent season, the Tigers play 16 games, with William playing in all of them. He rushes for 30 yards in the first game. On each succeeding game, he rushes for d more yards than in the previous game. He rushes for 120 more total yards in the last 6 games of the season than total yards in the first 10 games. Compute d.

4. Two lines l_1 and l_2 are perpendicular and intersect at point $(2,4)$ in the xy coordinate plane. A third line l_3 intersects l_1 at (a,a) and intersects l_2 at $(a+10, 2a+9)$. Suppose that $a<0$. Compute the area of a triangle whose vertices are the points that lie on exactly two lines out of l_1, l_2, and l_3.

5. In the distant galaxy of *Enchanted*, inhabitants use the base 6 number system. Lily and Madison have the same birthday and Lily is 15 *Enchanted* years older than Madison. After 10 *Enchanted* years, Lily's age, when represented in Earth's base 10 system, is now two times that of Madison's age in base 10. How old is Lily now in Earth's base 10 system?

6. In the addition problem below, each letter represents a different digit between 0 and 9. In how many distinct ways can the letters can be assigned with digits so that the equation is accurate?

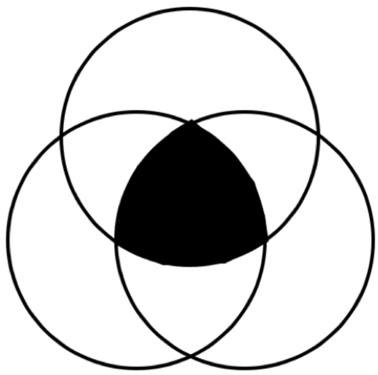
$$\begin{array}{cccc} & R & I & C \\ + & T & R & A \\ \hline R & A & M & C \end{array}$$

7. A virus breaks out on the faraway exoplanet of *Dystopia* in 2050. Suppose that 1 out of every 20 people in the population currently has the virus, which is asymptomatic. Currently, an antibody test is the only available method for detecting the virus. The antibody test will return positive 99% of the time for a person who has the virus. For a person who does not have the virus, the antibody test will accurately return negative 95% of the time and inaccurately return positive 5% of the time. A person from the population is randomly selected, tested, and their result is

positive. The probability that they actually have the virus can be expressed as $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute $p+q$.

8. Dr. Hu is standing at (0,0) in the coordinate plane and wants to travel to (6,6). She can move to the right or move up in one unit increments. However, there are three teleportation machines located at (1,5), (2,4), and (4,2). If she lands on a teleportation machine, she can choose to move normally (one unit to the right or upward) or she can teleport to another teleportation machine. Once she uses teleportation, all teleportation machines vanish and can no longer be used. Find the number of ways that she can travel from the origin to $(6,6)$.

9. Three congruent circles, each with radius 2 feet, are placed as shown such that each circle intersects with the centers of the other two circles. The area of the shaded region in feet squared, which is the area shared between all three of the circles, can be expressed as $m\pi + n\sqrt{q}$, where m, n, and q are positive integers and q is not divisible by the square of any prime. Compute $m+n+q$.



10. The first three terms of an arithmetic sequence are $8-8k$, $3k^2$, and k^4+2k^3 in that order. Additionally, k and 1 can be the first and second terms, respectively, of a convergent infinite geometric series. The sum of all possible values of k can be written in the form $a+b\sqrt{c}$, where a and b are integers, and c is a positive integer that is not divisible by the square of any prime. Compute $a^2+b^2+c^2$.