



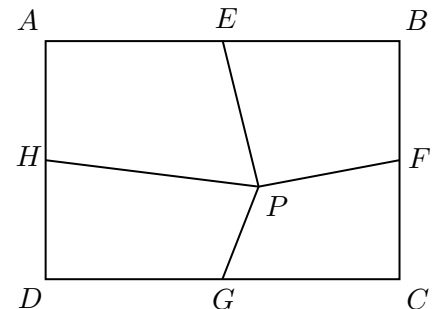
RAMC 2023

Middle School Tiebreaker Round

- **SCORING:** The questions in this round are used to break ties, and are not necessarily weighted the same.
- This round contains 10 questions to be solved in 30 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Do not flip the page until the proctor begins the round!

- Define the \star and \diamond operations as follows: $a \star b = a^2 - b$ and $a \diamond b = b^2 - a$. Evaluate $5 \star (3 \diamond 8)$.
- The first three terms in an arithmetic sequence are $2a$, $5a - 3$, and $6a + 2$, in that order, for some value of a . What is the 2023rd term in the sequence?
- Perry's family has 5 people in total. In a room, there are 11 red socks, 9 yellow socks, 7 orange socks, 5 green socks, 3 blue socks, and 1 purple sock. Each can be worn on either foot. What is the minimum number of socks that Perry must draw such that it is guaranteed that the family can all wear the same combination of not-necessarily-matching socks?
- Ethan is at a candy shop with 4 different flavors of identical candies. He has \$18 and candies cost \$2.50 each. He plans on spending as much of his money as possible on candies. How many unique ways can Ethan select candies to buy if he wants to have at least 1 candy of each flavor?
- Circles P and Q have diameters AB and CD , respectively, and they share a common tangent AD . Circle M is constructed, concentric to circle P , to pass through point D . Circle N is constructed, concentric to circle Q , to pass through point A . Given that $AB = 4CD$, the positive difference in area between circles M and N can be expressed as an integer multiple, k , of the area of circle Q . Find the value of k .
- How many permutations of the letters in the word TIEBREAKER have no consecutive vowels?
- The midpoints of the sides of rectangle $ABCD$ are connected to point P inside the rectangle, dividing the rectangle into four quadrilaterals as shown. Quadrilaterals $AEPH$, $BFPE$, and $CGPF$ have areas of 20, 18, and 15, respectively. Find the area of quadrilateral $DHPG$.



- Felix is playing a game. He rolls a 6-sided die numbered 1 to 6. If the number on top of the die is a prime number, he wins \$2. If it is not a prime number, he loses \$1. He continues rolling until he gets a 1. What is the expected number of dollars that Felix will gain from playing this game?
- Ben rolls two fair 100-sided dice, with faces numbered 1 to 100. The probability that the sum of the two rolls is at least 101 can be expressed as a ratio of relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.
- How many ways can the digits from 1 to 9 inclusive be arranged (each used once) to create 4 distinct numbers, each of which includes at least one digit? For example, four such numbers might be 7432, 15, 9, and 86.