RAMC 2021

Elementary I Individual Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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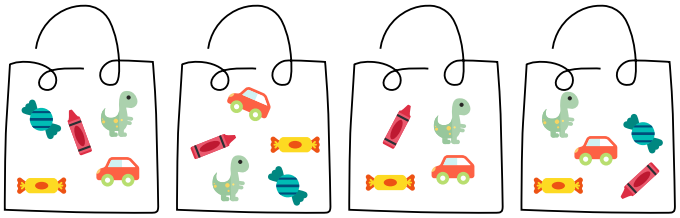
1. Lucy is filling gift bags with the same items in each bag. Which item is missing in one of the gift bags?

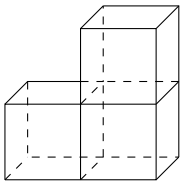


1.  2.  3.  4.  5. 

Answer: $\boxed{2}$

Solution: If we look carefully, every bag has 5 different items except for the 3rd bag which has 4 items. By comparing items in the 3rd bag with any other bag, you find that $\boxed{2}$, a piece of blue candy, is missing.

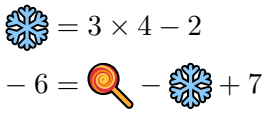
2. Yelena is building a small staircase with blocks. It takes three blocks to build two steps. How many blocks will it take her to build five steps?



Answer: $\boxed{15}$

Solution: We can see that the first step takes 1 block and adding the 2nd step adds 2 blocks. Thus, adding the 3rd step adds 3 blocks, the 4th step 4 blocks, etc. We will need to add 5 blocks for the 5th step. We need 5 steps so our final amount of blocks we need is $1+2+3+4+5=\boxed{15}$ blocks.

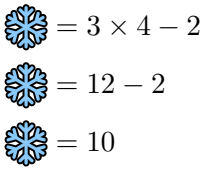
3. Toby placed stickers over some of the numbers in two problems of his math book, with the same stickers over the same numbers.



What number can be found under the sticker?

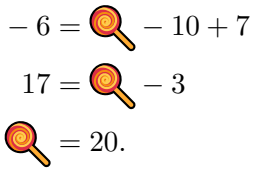
Answer: $\boxed{20}$

Solution: Because the 2nd equation requires the value of , we will use the first equation to solve for it.



We can replace with 10 in the 2nd equation to get

Evaluating, we end up with



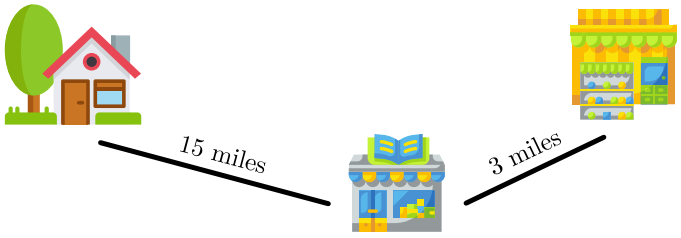
Thus, the number under the sticker is $\boxed{20}$.

4. Three identical full jugs of juice weigh 5 pounds in total. Three empty jugs weigh only 2 pounds in total. Richard has a full jug of juice, one half-full jug of juice, and an empty jug. If the weight of all three bottles can be expressed as a mixed number in simplest form, $a\,\frac{b}{c}$, find $a+b+c$.

Answer: $\boxed{6}$

Solution: To find the weight of the juice without the jug, subtract the weight of the empty jugs from the weight of the full jugs. Thus, the juice inside of the 3 jugs (excluding the jug itself) weighs $5-2=3$ pounds in total. Since there are 3 identical jugs, each one holds 1 pound of juice per jug.
Therefore, a full jug of juice is 1 pound, a half-full jug is 0.5 of a pound, and an empty jug has 0 pounds of juice. Adding the 2 pounds of the jug itself, we have a final weight of
$1+0.5+2=3.5$ pounds. We can write this as $3\,\frac{1}{2}$, to get our final answer of $3+1+2=\boxed{6}$.

5. Sally lives on the same street as the local bookstore and the local grocery store. The map below shows the distances between (from left to right) Sally's house, the book store, and the grocery store. Sally's day consists of going from her house to the book store, then returning from the bookstore, then going to the grocery store, and finally going back home. What total distance does Sally travel on this day, in miles?



Answer: $\boxed{66}$

Solution: The bookstore is 15 miles from Sally's house. It will take 15 miles to get there and 15 miles to come back, for a total of 30 miles. To get to the grocery store, she needs to travel to the grocery store, then the bookstore. This distance is $15 + 3 = 18$ miles. She repeats this distance to get home, for a total of 36 miles.
Adding both of her trips together yields $30 + 36 = \boxed{66}$ miles in total.

6. Maggie has 13 toys. Albert has 27 toys. How many toys does Albert have to give to Maggie in order for them to have an equal amount of toys?

Answer: $\boxed{7}$

Solution: The key here is to find the number of toys that each person will have after Albert gives his toys away. They have a total of $13+27 = 40$ toys. This means that each person will have 20 toys after the exchange. Albert starts with 27 toys, meaning that he needs to give Maggie $27 - 20 = \boxed{7}$ toys for each person to have an equal amount of toys.

7. The Rochester Forest Management (RFM) is planting trees in a recently developed area. They want to plant 150 trees in total. On the first day, they planted 80 trees. Each day after that, they planted half the amount of trees as the day before. How many days did it take for the RFM to finish planting all trees?

   Answer: $\boxed{4}$

   Solution: We list the number of trees planted on each day, with a running total.

day	# trees planted this day	total trees planted
1	80	80
2	40	120
3	20	140
4	10	150

   As we can see, it will take $\boxed{4}$ days for the RFM to finish planting.

8. How much does a pineapple cost in the figure below?

   Answer: $\boxed{6}$

   Solution: We notice that the 2nd expression has one extra tomato than the first. Since the price difference is \$4, we know that a tomato costs \$4. We can substitute this into the first expression, and see that a pineapple is $10 - 4 = \boxed{6}$ dollars.

9. Bob, Sally, Jason, Peggy, and April each choose a different number from the figure below. Jason's number is divisible by 3 and is larger than Sally's number. Bob's number is divisible by both 6 and 8. April's number is the closest to the the average of the 5 numbers (which is calculated by the sum of the five numbers, divided by 5). What number did Peggy choose?

Answer: $\boxed{32}$

Solution: Let us first figure out Bob's number. The only number that is divisible by 6 and 8 on this list is 24. Since Jason's number is divisible by 3, and his is larger than Sally's, his number is 27. April's number is the closest to the average, which is:

$$\frac{6+19+24+27+32}{5} = 21.6.$$

Thus, April's number is 19. We know that Sally's number is less than Jason's, so her number is 6. This only leaves one number left, $\boxed{32}$, which must be Peggy's number.

10. Arden has some coconuts. He divides his coconuts equally into 3 medium piles. He then divides each medium pile equally into 4 small piles. He then removes 1 coconut from each of the small piles. Each of the small piles now has 3 coconuts. How many coconuts did Arden have to start with?

Answer: $\boxed{48}$

Solution: Let us work backwards from the information we have. If each pile has 3 coconuts right now, then there must have been 4 coconuts in each pile before Arden removed one from each of the smallest piles. Now, recombine each small pile into a medium pile. The medium piles had 4 small piles, meaning that a medium pile has $4 \times 4 = 16$ coconuts.

Finally, we combine the medium piles to form the original pile of coconuts. There are 3 medium piles, meaning there are $3 \times 16 = \boxed{48}$ coconuts originally.

11. Sophie used letter blocks to spell out **MATH**. Some of the blocks were turned over. By rotating the "M" block two times clockwise, she can correct the letter M. If blocks have to be turned individually, how many times does Sophie need to rotate the blocks in this fashion for all the letters to be correct?

Answer: $\boxed{8}$

Solution: As shown in the figure, it will take 2 turns to rotate the letter M to its correct spot. We can do the same thing with all the other letters.

When we rotate letter A once, it will turn upside down. Turning it twice more will make it right side up. This operation is a total of 3 turns.

When we rotate letter T, it is already upside down, so we will only need to use 2 turns.

The letter H only requires one turn. Adding all the cases up, we have a total of $2 + 3 + 2 + 1 = \boxed{8}$ rotations to achieve our final result.

Note: The original image in this problem was incorrect, because the four blocks on the left that "spell out **MATH**" were already oriented correctly. In that case, the most accurate answer would be $\boxed{0}$.

12. The numbers 5000, 2201, and 1031 have digits that add up to 5. How many 4 digit numbers have digits that add up to 3? *Digits are allowed to repeat.*

Answer: $\boxed{10}$

Solution: Let us first consider the thousands digit. There are 3 cases, 3 as the thousands digit, 2 as the thousands digit, and 1 as the thousands digit.

Case 1: The thousands digit is 3. We know that since the sum of the 3 digits is 3, there is only one numbers: $3,000$.

Case 2: The thousands digit is 2. We know that one of the remaining 3 digits has to be 1, and the rest are 0. This leaves us with 3 numbers: 2100, 2010, and 2001.

Case 3: The thousands digit is 1. There are a few different cases to be considered here.

Sub-case 3a: The remaining three digits contain two 0s and one 2. We can put the 2 in 3 different spots, giving us 3 numbers: 1200, 1020, and 1002.

Sub-case 3b: The remaining three digits contain one 0 and two 1s. We can put the 0 in 3 different spots, giving us 3 numbers: 1011, 1101, and 1110.

In total, we have a total of $1 + 3 + 3 + 3 = \boxed{10}$ numbers that satisfy our condition.

13. Jack has a chocolate bar. He takes $\frac{1}{2}$ of the whole bar and gives it to Jill who takes $\frac{1}{2}$ of the remaining bar. Their friend Julian then takes $\frac{1}{4}$ of the remaining bar. The amount of the original chocolate bar that remains after Julian takes his piece can be expressed by the simplest fraction $\frac{p}{q}$. Find the sum $p+q$.

Answer: $\boxed{19}$

Solution: Jack begins by taking $\frac{1}{2}$ of the bar, leaving $\frac{1}{2}$ of the bar to Jill. Jill takes $\frac{1}{2}$ of the remaining bar, which is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the original bar. Jill passes on $\frac{1}{4}$ of the original bar to Julian. Julian takes $\frac{1}{4}$ of $\frac{1}{4}$, which he takes a total of $\frac{1}{16}$ of the original bar and leaves $\frac{3}{16}$ of the original bar. Thus, our final solution is $3+16=\boxed{19}$.

14. Alfredo the bird was out catching worms. On every 13^{th} attempt, he successfully catches a worm. In addition, on every 15^{th} attempt, his friend Benito gives him 3 more worms for free. How many worms will Alfredo have after exactly 600 attempts?

Answer: $\boxed{166}$

Solution: Let us first figure out how many times 15 goes into 600. We get 40. This is how many times he gets 3 worms from Benito, totaling up to 120.
He also gets one worm every 13^{th} attempt. 13 goes into 600 roughly 46 times. This means he will catch a work independently 46 times.
The total amount of worms he would catch by his 600^{th} attempt is $120+46=\boxed{166}$ worms.

15. Felix has 7 coins, 3 of which are currently heads up and 4 are tails up. In one move, Felix can use three fingers to flip any 3 coins to the other side at the same time. What is the least number of moves it takes for him to get all of the coins heads up?

Answer: $\boxed{2}$

Solution: Let us use the letter "H" to represent Heads and "T" for Tails. Felix starts with 3 H's and 4 T's, which we can write as **HHHTTTT**. In one move, we can flip two T's and one H to achieve a net gain of one Head. This leaves us with **HHHHTTT**. We can turn this into all heads in one more move, by turning the 3 T's all into Heads. Thus, the least amount of moves we need to flip all of them to heads is $\boxed{2}$ moves.