RAMC 2021

Elementary I Team Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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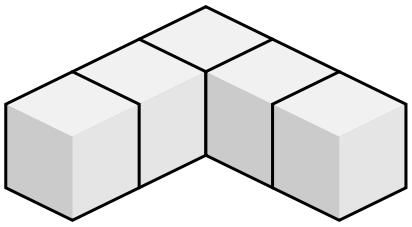
1. I leave the florist with 7 flowers. Someone outside hands me 135 more flowers. In return, I gave them 47 flowers. Now, how many flowers do I have?

   Answer: $\boxed{95}$

   Solution: After my first transaction, have 7 flowers. After the random stranger gives me his flowers, I now have $7 + 135 = 142$ flowers. Then, I give away 47 flowers to the stranger, leaving me left with $142 - 47 = \boxed{95}$ flowers.

2. Christine builds a structure with cubes, as shown. She will paint the entire structure blue, including the bottom. How many cube faces will be painted blue?



   Answer: $\boxed{22}$

   Solution: Let us count the sides facing out on all sides.
   First, we can see that there are 5 faces on the top.
   Next, we can see that there are 4 faces in the front, and 2 on the side, for a total of $4 + 2 = 6$.
   Now, we have to visualize how many are on the back and the bottom.
   We know that the top has the same amount as the bottom, so there are 5 faces there. We can also see that there are 6 faces on the back.
   Therefore, the number of faces that will be painted blue is $5 + 6 + 5 + 6 = \boxed{22}$ faces.

3. Joey and Joe need to fix 5 cars that have come to their Jacksonville Repair Shop. Each car is a different model, meaning that each has a different repair time. The times needed to fix each individual car are 60 minutes, 75 minutes, 45 minutes, 10 minutes, and 80 minutes. Both Joey and Joe only work on one car at a time but can not work on the same car together. However, they can work on different cars at the same time. What is the least amount of minutes that it will take them to fix all 5 cars?

Answer: $\boxed{135}$

Solution: We can notice that the most efficient way for them to fix all 5 cars is by not stopping as much as possible. We also notice that one of them will work on 3 cars, and the other one will work on 2 cars.

The sum of the times of all 5 cars is $60 + 75 + 45 + 10 + 80 = 270$. This means if we can get both of their work times close to $\frac{270}{2} = 135$, that will be the most efficient.

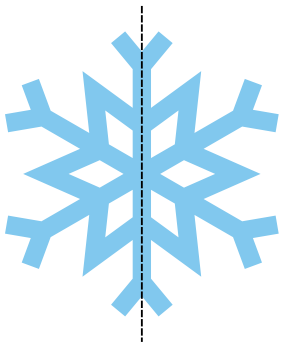
We notice that $60 + 75 = 135$ and $45 + 10 + 80 = 135$. From this, one should work on the 45, 10, and 80 minute cars, and the other one should work on the 60 and the 75 minute cars.

Therefore, the shortest possible time for them to finish all 5 cars is $\boxed{135}$ minutes.

We can also make a table representing their work:

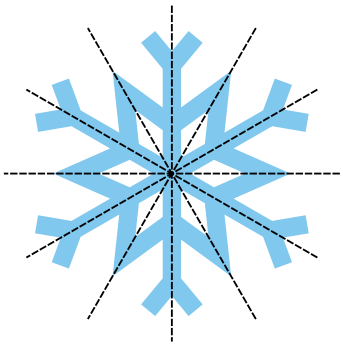
Time Since Start	**Joey**	**Joe**
Start	Begin working on 75 minute car	Begin working on 80 minute car
75 minutes in	Done. Start 60 minute car	5 minutes left
80 minutes in	55 minutes left	Done. Start 45 minute car
125 minutes in	10 minutes left	Done. Start 10 minute car
135 minutes in	Done Working	Done Working

4. A line of symmetry of the snowflake is shown. How many total lines of symmetry does the snowflake have?



Answer: $\boxed{6}$

Solution: A line of symmetry will divide the snowflake into two **identical** parts. As you can see in the problem, when you fold the snowflake over the line, it will match perfectly. We can draw lines through each pair of the snowflake's points and lines through the spaces between each pair of points.



From the figure, we see that there are $\boxed{6}$ lines of symmetry.

5. Denny and Jenny start jogging from the same point. Denny jogs 2 miles west, 1 mile south, 1 mile west, and finally 3 miles north. Jenny jogs 1 mile north, 2 miles east, and finally 1 mile north. Find the distance in miles between Denny and Jenny.

Answer: $\boxed{5}$

Solution: Let us look at where both Denny and Jenny are in relation to the starting point. Denny is 3 miles west and 2 miles north of the starting point. Jenny is 2 miles east and 2 miles north of the starting point.

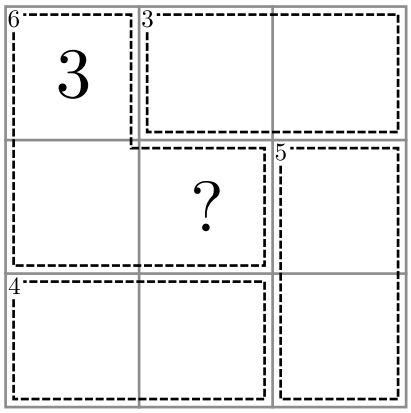
Since they are both 2 miles north from the starting point, we can draw a straight horizontal line between them. Jenny is 2 miles east of the starting point, and Denny is 3 miles west. Since East and West are opposite directions from each other, their total distance is given by $3 + 2 = \boxed{5}$ miles.

6. The captain of a Brazilian ship must choose 3 members to form his dive crew. How many different crews can he form if he has 5 members to choose from?

   Answer: $\boxed{10}$

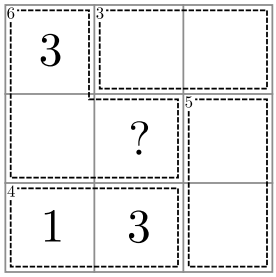
   Solution: The captain has 5 members to choose from to fill the first position, 4 to choose from to fill the 2^{nd} positions, 3 to choose from to fill the 3^{rd}. This gives us $5 \times 4 \times 3 = 60$ options. However, some of the options are repeated. For example, the above solution would count the diving crew "A, B, C" different from a crew with "C, A, B". To not over count, we must divide by the number of ways to arrange a crew of 3, which is $3 \times 2 = 6$. Thus, the number of diving crews possible is $60 \div 6 = \boxed{10}$.

7. The square shown below must be filled in such a way that each of the digits 1, 2, and 3 appear in each row and in each column. Numbers inside the enclosed areas surrounded by dotted lines must sum to the number indicated on that line. What is the number is represented by the question mark?

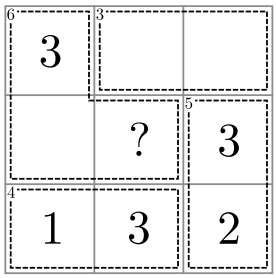


Answer: $\boxed{1}$

Solution: Let us first take a look at the two boxes that sum up to 4. Since we know that we can not put 2 of the same thing in the same row, the two numbers in these two boxes are 1 and 3. Since we can not put 2 of the same thing in the same column, we find that:



From here, we can fill in a 2 in the bottom right spot, and fill in a 3 directly above it, since those 2 squares must sum up to 5.



We can fill in the top-right spot with a 1, then fill the top-center spot with a 2. Therefore, the center spot must be a $\boxed{1}$, since the other two spots in the column are 2 and 3.

8. Let n be the sum of the first 1000 positive integers. Which choice best describes n?

   1. n is odd 2. n is even

   Answer: $\boxed{2}$

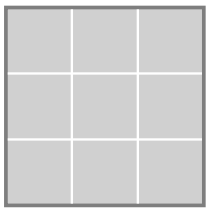
   Solution: Of the first 1000 numbers, we know that 500 of them are even and 500 are odd. Let us consider both of these, and then add the final sums together.

   Adding 500 even numbers together yields an even number.

   Adding 500 odd numbers, or an even number of odd numbers in general, yields another even number.

   Since the two sums are even, adding them together will yield another even number, meaning that the sum of the first 1000 numbers is even, and the answer is $\boxed{2}$.

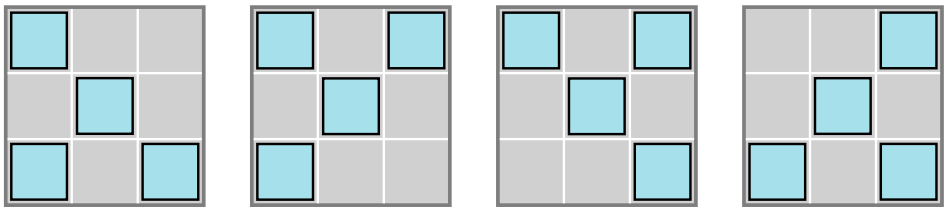
9. Leo wants to replace some of the square patches in his 9-patch quilt block with a blue fabric. Currently, all the patches in his quilt block are gray fabric. In how many ways can he replace 4 of the patches so that no two of the blue patches have sides that are touching each other?



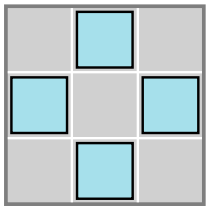
Answer: $\boxed{6}$

Solution: Let us consider a few cases:

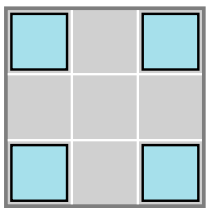
Case 1: There is a blue patch in the center. This means that 4 of the remaining 8 gray patches are available for use. (These are the corners.) There are 4 ways to choose the corner we will not replace. Therefore, there are 4 ways here.



Case 2: There is a blue patch in one of the side squares. This means that we have 5 squares to put 3 of them. However, all of these are touching each other, in a shape of a staple. This means there is only 1 way to make this work.



Case 3: There is a blue patch in one of the corners. There are 6 squares to put 3 blue ones in. However, since we already covered the case of one in the center, that one is blocked off. The remaining 5 form a shape of a continuous "L", meaning that there is only one way for this case.



Therefore, we have a total of $4 + 1 + 1 = \boxed{6}$ ways to satisfy our condition.

10. The Rochester Clock Tower casts a 30-foot shadow. At the same time, a light post casts a 60-inch shadow. The height of the light post is 12 feet. Find the height of the clock tower, in feet.

Answer: $\boxed{72}$

Solution: We first should notice that the light post casts a 60 inch = 5 foot shadow. The clock tower casts a shadow 6 times of the light tower. This means the clock tower is 6 times the height of the lamp post.

Since the lamp post is 12 feet tall, this means that the clock tower is $12 \times 6 = \boxed{72}$ feet tall.