RAMC 2021

Elementary II Individual Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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1. Evaluate the expression $2022 \div ((2 \times 0 + 3) \times 6 + 0 - 6 \times 2 - 0)$.

   Answer: $\boxed{337}$

   Solution: We first evaluate inside of the second parenthesis to get

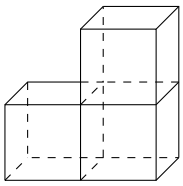
   $$2022 \div (3 \times 6 + 0 - 6 \times 2 - 0).$$

   Next, we evaluate inside of the parenthesis to get

   $$2022 \div 6.$$

   This is equal to $\boxed{337}$.

2. Yelena is building a small staircase with blocks. It takes three blocks to build two steps. How many blocks will it take her to build nine steps?



   Answer: $\boxed{45}$

   Solution: We can see that the first step takes 1 block and the 2^{nd} step takes 2 blocks. Thus, the 3^{rd} step will take 3 blocks, the 4^{th} step 4 blocks, etc. We will need 9 blocks for the 9^{th} step. We need 9 steps so our final amount of blocks we need is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = \boxed{45}$ blocks.

3. How much does a pineapple cost in the figure below, in dollars?

   Answer: $\boxed{6}$

   Solution: We notice that the right group of fruits has one extra tomato than the left. Since the price difference is \$4, we know that a tomato costs \$4. We can substitute this into the left group, and see that a pineapple is $10 - 4 = \boxed{6}$ dollars.

4. Jack has a chocolate bar. He takes half of the whole bar and gives it to Jill, who takes $\frac{1}{4}$ of the remaining bar. Their friend Julian then takes $\frac{1}{5}$ of the remaining bar. If the amount of the original chocolate that bar remains after Julian takes his piece is $\frac{p}{q}$, find the sum $p+q$.

   Answer: $\boxed{13}$

   Solution: Jack begins by taking $\frac{1}{2}$ of the bar, leaving $\frac{1}{2}$ of the bar to Jill. Jill takes $\frac{1}{4}$ of the remaining bar, which is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ of the original bar. Jill passes on $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ of the original bar to Julian. Julian takes $\frac{1}{5}$ of $\frac{3}{8}$th of the chocolate bar, which is a total of $\frac{3}{40}$ of the original bar and leaves $\frac{3}{8} - \frac{3}{40} = \frac{3}{10}$ of the original bar. Thus, our final solution is $3 + 10 = \boxed{13}$.

5. Alexander wants to buy candy that costs 66 cents. He only has quarters, nickels, and pennies. What is the least number of coins he can use to pay for the candy?

   Answer: $\boxed{6}$

   Solution: We want to maximize the amount of larger coins. This is because 1 quarter = 5 nickels = 25 pennies. We can fit 2 quarters into 66 cents. We have 16 cents remaining. We can now use 3 nickels and 1 penny to complete 16 cents. Therefore, we have $2 + 3 + 1 = \boxed{6}$ coins.

6. Jenny is trying to tile her kitchen with square tiles that each have an side length of 6 inches. The kitchen is 3 feet by 3 feet, but there is a counter which is 1 foot by 6 inches. If Jenny does not need to tile the area under the counter, how many tiles will she need to tile the entire kitchen?

   Answer: $\boxed{34}$

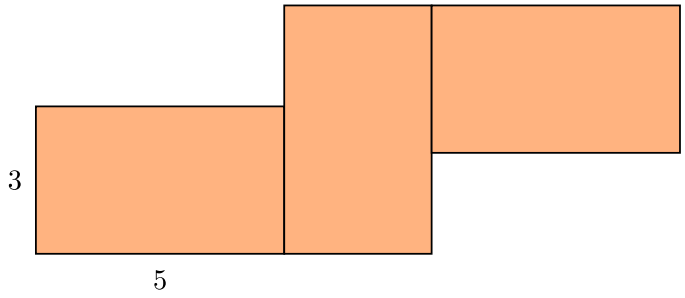
   Solution: Let us first convert all measurements to inches to remove confusion. Since there is 12 inches per foot, we know that the whole kitchen is 36 inches by 36 inches. Without the counter, we can fit 6 tiles per side, for a total of $6 \times 6 = 36$ tiles.
   The kitchen counter is 12 inches by 6 inches, which fits exactly 2 tiles. Therefore, we need to use $\boxed{34}$ tiles to cover the floor.

7. Alfredo the bird was out catching worms. He catches a worm on every 13^{th} attempt. In addition, for every 15^{th} attempt, his friend Benito gives him 3 more worms. How many worms will Alfredo have after exactly 600 attempts?

   Answer: $\boxed{166}$

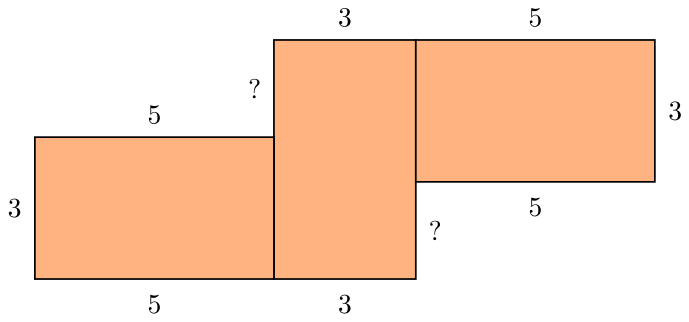
   Solution: Let us first figure out how many times 15 goes into 600. We get 40. This is how many times he gets 3 worms from Benito, totaling up to 120.
   He also gets one worm every 13^{th} attempt. 13 goes into 600 roughly 46 times. This means he will catch a work independently 46 times.
   The total amount of worms he would catch by his 600^{th} attempt is $120 + 46 = \boxed{166}$ worms.

8. Michael has three rectangles, all of the same size. He makes the figure below with his rectangles. What is the perimeter of the figure?

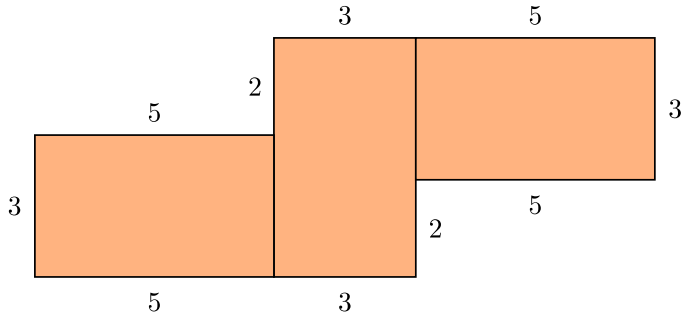


Answer: $\boxed{36}$

Solution: Label the long and short sides of the rectangles on the edge of the figure with 5 and 3.



The unknown lengths can be found by subtracting the short side length of the rectangle from the long side length. We can evaluate them as $5 - 3 = 2$.



Now, we sum all the lengths to find the perimeter.

$$\begin{aligned} \text{Perimeter} &= 2 \times (5 + 3 + 2 + 5 + 3) \\ &= 2 \times 18 \\ &= 36 \end{aligned}$$

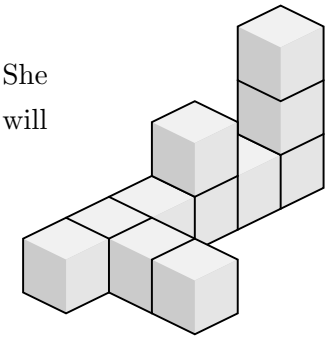
Therefore, the perimeter is $\boxed{36}$.

9. There are 18 pieces of candy in Richard's bucket. 12 are chocolates, 3 are lollipops, 2 are caramels, and 1 is a piece of fudge. Richard chooses 3 candies from the bucket, without replacing them after he takes them out. If the probability that Richard chooses a lollipop, a chocolate, and the piece of fudge, in that order is $\frac{p}{q}$, where p and q are relatively prime positive integers, find $p+q$.

Answer: $\boxed{137}$

Solution: The probability of choosing the lollipop first is $\frac{3}{18} = \frac{1}{6}$.
After Richard chooses the lollipop, there are now 17 pieces of candy, 12 of which are chocolate. This giving us a probability of $\frac{12}{17}$.
For the fudge, there are 16 pieces of candy left, with only 1 being fudge. This leaves us with a probability of $\frac{1}{16}$.
This gives us a total probability of $\frac{1}{6} \times \frac{12}{17} \times \frac{1}{16} = \frac{1}{136}$. Thus, our answer is $1 + 136 = \boxed{137}$.

10. Natalie builds a structure with 11 cubes, as shown in the figure to the right. She paints the entire structure green, including the bottom. How many cubes will have at least 3 of their faces painted green?



Answer: $\boxed{11}$

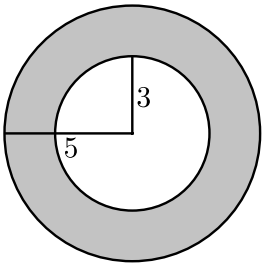
Solution: We notice that in order for at least 3 of their faces to be painted green, the cube has to border at most 3 cubes. We see that all cubes in the diagram border at most 3 cubes, meaning that all $\boxed{11}$ cubes will have at least 3 faces painted green.

11. Felix has 7 coins, 3 of which are currently heads up and 4 are tails up. In one move, Felix can use three fingers to flip any 3 coins to the other side at the same time. What is the least number of moves it takes for him to get all of the coins heads up?

Answer: $\boxed{2}$

Solution: Let us use the letter "H" to represent Heads and "T" for Tails. Felix starts with 3 H's and 4 T's, which we can write as **HHHTTTT**. In one move, we can flip two T's and one H to achieve a net gain of one Head. This leaves us with **HHHHTTT**. We can turn this into all heads in one more move, by turning the 3 T's all into Heads. Thus, the least amount of moves we need to flip all of them to heads is $\boxed{2}$ moves.

12. Bob has two concentric circles, one with radius 5 and one with radius 3. The area of the region inside the larger circle but outside the smaller circle can be expressed as $a\pi$. Find a.



Answer: $\boxed{16}$

Solution: We know that the area of a circle is $\pi \times r^2$ where r is the radius. To find the area between the circles, find the difference between the large and the small one. The area of the large circle is $\pi \times 5^2 = 25\pi$ and the small circle is $\pi \times 3^2 = 9\pi$. The difference is $25\pi - 9\pi = 16\pi$, so our answer is $\boxed{16}$.

13. There are only chickens and pigs in Stephen's barn. If Julia counts 28 heads and 80 feet, how many pigs are in the barn?

Answer: $\boxed{12}$

Solution: Firstly, we know that chickens have 2 legs and 1 head, and pigs have 4 feet and 1 head. Let c be the number of chickens, and p be the number of pigs. We can set up a system of equations.

$$c + p = 28$$
$$2c + 4p = 80$$

Multiply the first equation to get $2c + 2p = 56$, and subtract it from the second equation. We get $2p = 24 \implies p = 12$. Therefore, there are $\boxed{12}$ pigs.

14. John draws 8 points on a sheet of paper, such that no 3 points can lie on the same line. How many lines can he draw that go through exactly 2 points?

Answer: $\boxed{28}$

Solution: There are 8 choices for the first point, and 7 for the second. Each of these has 2 cases, so we need to divide by 2 (e.g. line AB and BA are both counted, even they are the same line). Since no three points are collinear, we know that we are not over counting on that. Thus, there are $\boxed{28}$ lines that can be drawn.

15. What is the units digit of 2022^{2021}?

Answer: $\boxed{2}$

Solution: The units digit of a product depends only on the product of the units digits of the multiplicands, so we can use the expression 2^{2021} instead.
The powers of 2 follow a pattern that we can use to figure out the units digit of 2^{2021}.

n	0	1	2	3	4	5	6	7	8
2^n	1	2	4	8	16	32	64	128	256
units digit of 2^n	1	2	4	8	6	2	4	8	6

We can see a pattern, starting from $n = 1$, the units digit goes 2, 4, 8, 6, and repeats. Since 2020 is a multiple of 4, we know that 2021 has a remainder of 1, meaning that its units digit is $\boxed{2}$.