RAMC 2021

Elementary II Team Solutions

Contest Problems/Solutions proposed by the Rochester Math Club problem writing committee:

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1. Which of the shapes below has the largest interior angle?

   1. Regular Pentagon 2. Regular Hexagon 3. Square 4. Regular Octagon 5. Rectangle

   Answer: $\boxed{4}$

   Solution: The formula used to calculate the sum of all interior angle is $I = (S - 2) \times 180$, where S is the number of sides and I is the sum of all interior angles. Let us find the interior angles of all 5 choices.

   We notice that all angles in each individual shape are equal. This will make it easier.

   The Regular Pentagon has 5 sides, so $I = 3 \times 180 = 540$. Each interior angle, therefore, is $\frac{540}{5} = 108$ degrees.

   The Regular Hexagon has 6 sides, so $I = 4 \times 180 = 720$. Each interior angle, therefore, is $\frac{720}{6} = 120$ degrees.

   The Square has 4 sides, so $I = 2 \times 180 = 360$. Each interior angle is $\frac{360}{4} = 90$ degrees.

   The Regular Octagon has 8 sides, so $I = 6 \times 180 = 1080$. Each interior angle, therefore, is $\frac{1080}{8} = 135$ degrees.

   The Rectangle, like the square, has 4 sides, so it has interior angles of 90 degrees.

   Therefore, $\boxed{4}$, the Regular Octagon has the largest interior angle.

2. Tracy needs to buy some merchandise to sell at her store. She buys two carts that are \$50 each, three suitcases that are \$30 each, and a lamp that is \$10. The store Tracy is shopping at currently has a 25% discount on all items. Tracy also was the 100^{th} customer that day, so she gets an additional 50% off her purchase. If Tracy plans to sell every piece of merchandise at \$30 each, what is her profit, in dollars, after she sells all 6 items?

   Answer: $\boxed{105}$

   Solution: Let us first figure out how much Tracy spends for the merchandise. The total price is 2 carts ($\$50 \times 2 = \$100$), 3 suitcases ($\$30 \times 3 = \$90$), and a lamp (\$10). This totals to \$200.

   Applying discounts, we first apply the 25% discount. This gives us a price of $\$200 - (25\%) \times \$200 = \$150$.

   The 50% discount gives us the entire thing for 50%, giving us her total cost of $\$150 - (50\%) \times \$150 = \$75$.

   Tracy sells each item for a $\$30 \times 6 = 180$ gain, meaning that her profit is $\$180 - \$75 = \boxed{105}$ dollars.

3. Harry is thinking of a number. Harry says that his number is a two-digit prime number, whose units digit is less than twice the tens digit. He also says that if you add 36 to his number, you would get his number, but digits reversed. What is the number that Harry is thinking about?

   Answer: $\boxed{59}$

   Solution: We have a few observations we notice:

   1. This number is prime. (Given)
   2. This number's units digit is less than twice of its tens digit. (Given)
   3. This number is less than 64 as $64 + 36 =$ a 3 digit number.
   4. The difference between the two digits of this number is 4. This is because $10a + b - (10b + a) = 9a - 9b = 9(a - b) = 36$.

   We can use the 3$^{\text{rd}}$ and 4$^{\text{th}}$ observations to list out a few values: 15, 26, 37, 48, 59, 51, and 62. Here, our only values that are prime are 37 and 59. But for 37, twice the tens digit is $3 \times 2 = 6 < 7$, meaning that our only solution is $\boxed{59}$.

4. The Rochester Clock Tower casts a 30-foot shadow. At the same time, a light post casts a 60-inch shadow. The height of the light post is 12 feet. Find the height of the clock tower, in feet.

   Answer: $\boxed{72}$

   Solution: We first should notice that the light post casts a 60 inch = 5 foot shadow. The clock tower casts a shadow 6 times of the light tower. This means the clock tower is 6 times the height of the lamp post.
   Since the lamp post is 12 feet tall, this means that the clock tower is $12 \times 6 = \boxed{72}$ feet tall.

5. Sarah's salary in 1999 was \$200. In the beginning of 2000, her salary increased by 20% and in the beginning of 2001, her salary decreased by 20%. What is Sarah's salary, in dollars, at the end of 2001?

   Answer: $\boxed{192}$

   Solution: We can just calculate our values in 2000 and 2001.
   In 2000, Sarah has an increase of salary by 20%, and therefore has a salary of $\$200 + (20\%) \times \$200 = \$200 + \$40 = \$240$.
   In 2001, Sarah has a decrease of salary by 20%. Her salary is calculated by $\$240 - (20\%) \times \$240 = \$240 - \$48 = \boxed{192}$ dollars.

6. Jason the race car driver is training for his next race. His training routine consists of driving on a straight 10 mile race track two times. The first time he goes on the track, he drives at a constant speed of 100 miles per hour. The second time, he gets an engine upgrade, and he is able to drive at a constant speed of 150 miles per hour. What is Jason's average speed, in miles per hour, for those two runs?

   Answer: $\boxed{120}$

   Solution: We should use the distance = rate × time formula. Our distance is 10 miles $\times 2 = 20$ miles. We can calculate the time he was on the track with the same formula.

$$10 \text{ miles} = 100 \, \frac{\text{miles}}{\text{hour}} \times x \text{ hours}$$
$$10 \text{ miles} = 100 \, \frac{\text{miles}}{\text{hour}} \times \frac{1}{10} \text{ hours.}$$

   For the second lap,

$$10 \text{ miles} = 150 \, \frac{\text{miles}}{\text{hour}} \times x \text{ hours}$$
$$10 \text{ miles} = 150 \, \frac{\text{miles}}{\text{hour}} \times \frac{1}{15} \text{ hours.}$$

   The total time is $\frac{1}{10} + \frac{1}{15} = \frac{1}{6}$ of an hour. Substituting it back into the first equation, we have

$$20 \text{ miles} = y \, \frac{\text{miles}}{\text{hour}} \times \frac{1}{6} \text{ hours}$$
$$20 \times 6 \, \frac{\text{miles}}{\text{hours}} = y \, \frac{\text{miles}}{\text{hour}}$$
$$y = 120.$$

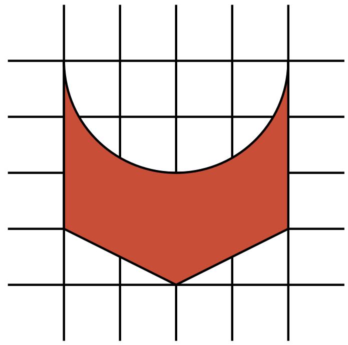
   Thus, Jason's average speed was $\boxed{120}$ miles per hour.

7. When Natalie divides a number by 8, the remainder is 7. What is the remainder when Natalie divides three times the original number by 8?

   Answer: $\boxed{5}$

   Solution: Natalie's number must be 7 greater than any multiple of 8. We can write this number as $8k + 7$ where k is an integer. When we multiply this by 3, we get $3 \times (8k + 7) = 8(3k) + 21$. We know that $8(3k)$ is still a multiple of 8, so we have a remainder of 21. But, since we can write 21 as $8 \times 2 + 5$, we know that the remainder is $\boxed{5}$.

8. The grid below is composed of unit squares. If the area of the shaded region can be expressed as $a - b\pi$, find $a + b$.



Answer: $\boxed{16}$

Solution: We notice that the shaded area is a 4 by 4 square missing a semicircle and 2 identical triangles.

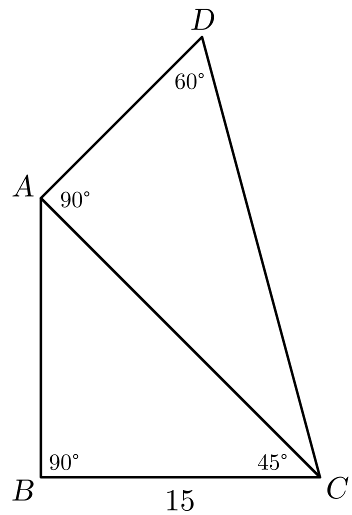
To find the area of the semicircle, we see that the diameter is 4 units long, meaning that the radius is 2 units. Thus, the area of the semicircle is

$$\begin{aligned}\text{Area of the semicircle} &= \frac{1}{2} \times \pi \times r^2 \\ &= \frac{1}{2} \times \pi \times 4 \\ &= 2\pi.\end{aligned}$$

The area of one of the triangles is $\frac{1}{2} \times 1 \times 2 = 1$. We have 2 of these, for an area of 2.

Since the area of a 4 by 4 square is 16, the area of the shaded area is $16 - 2 - 2\pi = 14 - 2\pi$, giving us a final answer of $14 + 2 = \boxed{16}$.

9. If the length of side CD is $a\sqrt{b}$ in simplest form, find $a \times b$.



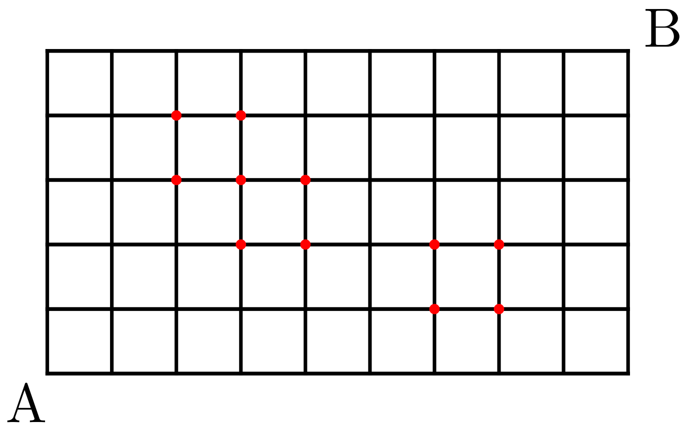
Answer: $\boxed{60}$

Solution: The problem has 2 special right triangles, a 45-45-90 and a 30-60-90 right triangles. Applying the property of 45-45-90 triangles gives us $AC = 15\sqrt{2}$. Using the property of the 30-60-90 triangle, this gives us $CD = 15\sqrt{2} \times \frac{2}{\sqrt{3}} = \frac{30\sqrt{2}}{\sqrt{3}}$.
However, we need to make it the so that our value is in simplest radical form. We need to rationalize the denominator by multiplying both the numerator and the denominator by $\sqrt{3}$.

$$\begin{aligned}\frac{30\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} &= \frac{30\sqrt{6}}{\sqrt{9}} \\ &= \frac{30\sqrt{6}}{3} \\ &= 10\sqrt{6}\end{aligned}$$

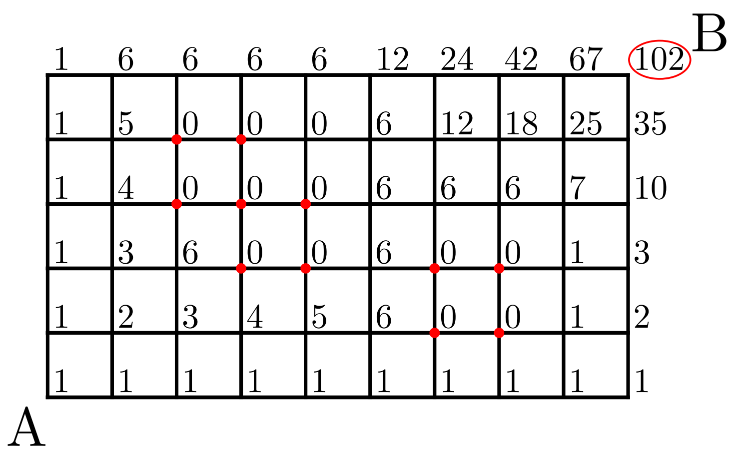
Therefore, our final answer is $10 \times 6 = \boxed{60}$.

10. How many ways can you move along the grid from point A to point B, if you can only move up and to the right, and you can not visit a point marked with a red dot?



Answer: 102

Solution: We can use the waterfall method. Every lattice point is labeled with the number of paths there are from point A.



As we can see, there are 102 paths.