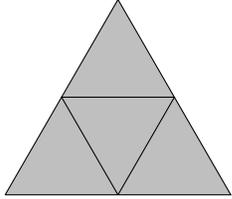


RAMC 2021

Middle School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each.
- This round contains 15 questions to be solved in 45 minutes. Problems towards the end tend to be more difficult than problems toward the beginning.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- All answers are integers. When submitting answers, do not add additional characters (such as spaces or units) beyond pure numerical digits, with the exception of a minus (-) symbol when needed.
- If you believe there is an error on the test, submit a challenge to rochestermathclub@gmail.com. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Take a moment to check that your information is entered correctly!

1. Johnny needs to buy 22 sodas. The store sells sodas in packs of 10, 5, and 1. The 10-pack costs \$15.20, the 5-pack costs \$7.30, and the 1-pack costs \$1.50. If Johnny wants to spend the least amount of money to , how many packs will he buy?
 2. Let $a\Omega b = a^2 - 2ab$ and let $a\Phi b = b^2 - 2a$. What is the value of $((25\Omega 2) - (3\Phi 5))\Phi 2$?
 3. There are 120 students at a graduation ceremony. It takes 5 seconds for each student to walk across the stage. As the students begin walking across the stage, Mr. Lee's digital watch reads 01:00 PM. What is the sum of the digits on Mr. Lee's watch after the last student walks across the stage?
 4. The figure to the right consists of four equilateral triangles of side length s . The height of the entire figure is 24. The value s can be expressed as $a\sqrt{b}$, where a is an integer and b is not a multiple of any perfect square larger than 1. Find $a + b$.
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5. A car's instantaneous velocity, in meters per second, is given by the function $f(x) = x^2 + 5$, where x is the number of seconds after the car begins moving. What is the car's instantaneous velocity in *kilometers per second* when 2 minutes have passed? Round your answer to the nearest integer.
 6. Find the largest prime divisor of $7^6 - 1$.
 7. Jack has 9 coins, consisting of only quarters, dimes, nickels, and pennies, with a total value of \$1.31. He has as many quarters as nickels and dimes combined, and he has half as many pennies as dimes. How many nickels does Jack have?
 8. How many positive integer factors does 2520 have?
 9. Convert 3120_4 to base 14. Do not include subscript base notation in your answer.
 10. One cold winter night, snow begins falling at a rate of 2 inches per hour, and Nathan begins shoveling his 20-foot by 20-foot driveway. He shovels and removes 25 square feet of snow every hour, regardless of the height of snow, and he always maximizes the volume of snow he removes. What is the volume of snow (in cubic feet) that is on Nathan's driveway after he shovels for 7 hours?
 11. If Bobby can eat a pizza in 30 minutes and Cassandra can eat a pizza in 15 minutes, how long, in minutes, will it take for Bobby and Cassandra to eat a pizza together?

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12. Jacob writes N , a three-digit multiple of 7, on a whiteboard, then covers one of the digits with a piece of paper. Julie knows that N is a multiple of 7. No matter which of the three digits are being covered, if Julie looks at the board, she can be certain of the value of the digit that is being covered. How many possible values of N are there?
13. Two cubes with integer side lengths have a combined volume of 7110. Find the sum of the possible sums of their side lengths.
14. A police detective has an 80% chance of solving any crime. The detective is given 5 crimes to solve. The probability that the detective solves exactly 3 of the 5 crimes can be expressed as a ratio of two relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.
15. Roots r_1 , r_2 , and r_3 of the polynomial $x^3 - 5x^2 - 8x + a$ satisfy the equation $r_1 + 2r_2 + 4r_3 = 0$. What is the sum of the possible values of a ?