

RAMC 2021

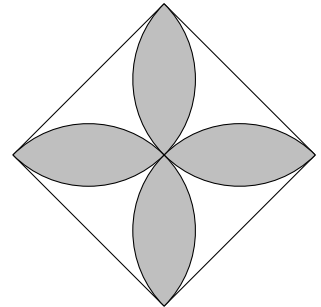
Middle School Team Round

- **SCORING:** The first 5 questions are worth 2 points each, and last 5 questions are worth 3 points each.
- This round contains 10 questions to be solved in 25 minutes. Problems towards the end tend to be more difficult than problems toward the beginning.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- All answers are integers. When submitting answers, do not add additional characters (such as spaces or units) beyond pure numerical digits, with the exception of a minus (-) symbol when needed.
- If you believe there is an error on the test, submit a challenge to rochestermathclub@gmail.com. Please include your name, level (Elem I/II, MS, HS), and explanation of the problem and your solution.

Take a moment to check that your information is entered correctly!

1. Pam has a cylindrical measuring cup with a diameter of 10 cm. The water level in the cup is currently at 4 cm. When Pam completely submerges three rocks of equal volume into the cup, the water level rises to 7 cm. If the volume of one of the rocks can be written as $a\pi \text{ cm}^3$, what is a ?
2. Suppose that in a certain state, license plate numbers consist of four unique characters, two digits followed by two letters. George wants a license plate number where the two digits are in increasing numeric order, and the two letters on his plate are in alphabetical order (e.g. 07AC is valid, but 22GA is not). How many license plate numbers can George choose from?
3. A box contains 17 blue balls, 15 red balls, 11 pink balls, 7 green balls, and 5 yellow balls. What is the least number of balls that must be drawn from the box without replacement to guarantee that at least 10 balls of the same color will be drawn?
4. The 6-digit number $789ABC$ is divisible by 7, 8, and 9, and all of its digits are distinct. What is the 3-digit number ABC ?
5. Sally and Ella are playing a guessing game. Sally's number is n . She tells Ella that n and 50 have a greatest common factor of 5, and that n and 175 have a least common multiple of 1925. What is the sum of all possible values of n ?
6. For a number N in base 10, the following holds true: $N_{10} = 763_b = 394_{b+5}$, where b is a positive integer. What is N ?

7. Four overlapping semi-circles are constructed inside a square of side length 12, with the sides as diameters, as shown. The area occupied by at least 2 semi-circles (shaded gray) can be expressed as $a\pi + b$. Find $a + b$.



8. The rule for divisibility by 109 is given by the following: take a positive integer, multiply the last digit by some positive integer k , then add it to the number formed by removing the last digit; if the result is divisible by 109, so is the original integer. Find the smallest value of k such that this is true.
9. The sum $\frac{1}{60} + \frac{1}{84} + \frac{1}{112} + \frac{1}{144} + \frac{1}{180} + \frac{1}{220} + \frac{1}{264} + \frac{1}{312} + \frac{1}{264} + \frac{1}{420}$ can be written as a ratio of two relatively prime positive integers, $\frac{a}{b}$. Find $a + b$.

10. Cory's favorite number is a positive integer between 1 and 10000, inclusive. Lucas wants to figure out what this number is, and plays the following game: Lucas picks a number n between 1 and 100, inclusive, and asks Cory for the remainder when his favorite number is divided by n . Lucas repeats this until he knows for certain the value of Cory's favorite number. Let m be the least number of values Lucas must ask Cory about to figure out Cory's favorite number. Given that Lucas uses exactly m values, determine the smallest possible sum of the values of n that Lucas uses.