RAMC 2024

High School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each, for a total of 20 possible points.
- This round contains 15 questions to be solved in 45 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (E1/E2/MS/HS), and your solution to the problem with explanation.

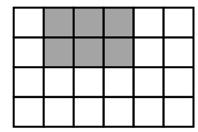
Do not flip the page until the proctor begins the round!

1. Evaluate $11^2 + 22^2 + 33^2 + 44^2 + 55^2$.

2. Positive integers a and b satisfy $a < b$. If a has 3 positive integer factors, and b has 5, what is the smallest possible value of $b - a$?

3. Four of the internal angles in a convex pentagon measure x degrees. The set of possible values of x is exactly the interval $a < x < b$. Find $a + b$.

4. Consider a 4 by 6 grid of unit squares. How many ways are there to color a rectangle consisting of some of these unit squares such that at least 7 squares remain uncolored? One such coloring is shown to the right.



5. Let $f(x)$ be a quadratic function such that $f(-3) = 8$ and $f(3) = 2$. If $f(x)$ intersects the x-axis at exactly one point, find the sum of the possible x-intercepts of $f(x)$.

6. Let $ABCDEF$ be a regular hexagon with side length 2 and center G. The sum of the areas of all nondegenerate triangles formed by any 3 of these 7 points can be expressed as $k\sqrt{3}$. Find k.

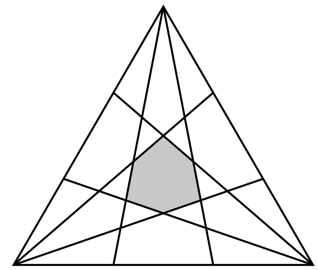
7. Asha writes 101 numbers on a board in a line, each a number selected uniformly at random from the set $\{-2, -1, 0, 1, 2\}$. She then makes a new line of 100 numbers below the first row, which contains the absolute value of the difference between consecutive numbers. For example, if the first row contains the numbers, $-2, 1, -1$, then the second row contains $3, 2$. She then adds all of the numbers in the second row. What is the expected value of this sum?

8. Alvin has 8 unique books: two number theory books, two algebra books, two geometry books, and two combinatorics books. How many ways can he order these books on his shelf such that neither geometry book is the leftmost book and both combinatorics books are to the right of both number theory books?

9. Alice is climbing an escalator, and Bob is climbing a flight of stairs of the same length. Due to the other people on the escalator, Alice's speed, relative to the escalator steps, is only $\frac{1}{4}$ of her normal walking speed. Alice and Bob start climbing at the same time. Bob reaches the top 5 seconds before Alice, at which point Alice is 75% of the way up. Given that they have the same normal walking speed, by how many seconds would Alice beat Bob to the top if there were no other people on the escalator slowing her down?

10. Kevin is hanging up banners on a clothesline. He has 8 differently colored banners: white, red, orange, yellow, green, blue, purple, and black. The white, red, and orange banners must hang to the left of all other banners, and the purple and black banners must hang to the right of all other banners. If Kevin can hang up any number of banners, but must hang up at least one banner, how many ways can he do so?

11. From each vertex of an equilateral triangle, two cevians are drawn that trisect the opposite side, as shown. The shaded hexagon comprises $\frac{m}{n}$ of the total area of the triangle. If m and n are relatively prime positive integers, find $m+n$.



12. Suppose that the equation

$$a^2 + y^2 = (x-6)^2 + (b-9)^2 = (a-b)^2 + (x-y)^2 = k$$

has exactly one solution (a, b, x, y) for given a value of k. What is k?

13. Let # be the concatenation function, where $a\#b$ is the number you get by placing a and b next to each other in the same order; e.g. $14\#34 = 1434$. Consider the sequence $a_k = (k+1)\#(3k)$ for $k \geq 1$. Find the smallest number in this sequence that is divisible by another number in the sequence.

14. For a positive integer n, the value $2^n - n$ is divisible by 13. Find the 25th smallest possible value of n.

15. Points A and B lie on the edge of a circular piece of paper such that $\widehat{AB} = 42°$. The paper is then folded along chords m and n, bringing A and B together to meet at point C, such that after folding, the new arcs are tangent at C. The extensions of m and n intersect to meet at an acute angle α. What is the maximum possible degree measure of α?