RAMC 2024

High School Invitational Round

- **SCORING:** The questions in this round are all worth 1 point each, for a total of 10 points.
- This round contains 10 questions to be solved in 30 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (E1/E2/MS/HS), and your solution to the problem with explanation.

Do not flip the page until the proctor begins the round!

1. Silas has 10 clean socks in his drawer. He wears a pair of them, then accidentally puts the dirty socks back into the drawer. Quickly, he rummages through the drawer, randomly retrieving 4 socks. The probability that he recovers both dirty socks is $\frac{p}{q}$ for relatively prime positive integers p, q. Find $p+q$.

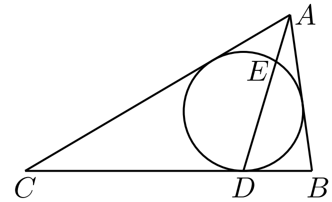
2. Given that a, b, and c are digits, find the three-digit integer $\underline{a}\,\underline{b}\,\underline{c}$ that satisfies

$$15! = 1{,}3\,0\,7{,}6\,7\,4{,}\underline{a}\,\underline{b}\,\underline{c}{,}0\,0\,0$$

3. Consider a sequence with $a_0 = 0$, $a_1 = 1$, and $a_n = 18a_{n-1} + 1008a_{n-2}$ for $n \geq 2$. As n approaches infinity, the ratio $\frac{a_{n+1}}{a_n}$ approaches some value r. What is r?

4. Find the number of pairs of integers (x, y) that satisfy $x^2 + y^2 < 81$.

5. The incircle of triangle ABC is tangent to side BC at point D, as shown. Segment AD intersects the incircle again at E, such that $AE : DE = 1 : 3$. The length of AD can be expressed as $\frac{m}{n}$, for relatively prime positive integers m and n. Given that $BD = 14$ and $CD = 34$, find $m+n$.



6. How many permutations of "FOLKLORE" contain a palindrome with length at least 2? For example, "FLEROLOK" is one such permutation, since "OLO" is a palindrome of length 3.

7. Determine the smallest positive value of x that satisfies

$$\cos(5^\circ) + \cos(25^\circ) + \cos(65^\circ) + \cos(85^\circ) = \sqrt{6}\cos(x^\circ).$$

8. Find the sum of all possible x, where $0 \leq x < 5 \cdot 7 \cdot 11$, that satisfy

$$x^2 \equiv 4 \bmod 5, \quad x^2 \equiv 4 \bmod 7, \quad \text{and } x^2 \equiv 4 \bmod 11.$$

9. Pyxis, the point, is wandering around the xy-plane. He starts at the origin facing the positive x direction. For his first step, he moves 5 units forward and then rotates counterclockwise by an angle $\theta = \arctan(3/4)$. For every future step, he moves forward five-eighths the length of the previous step and then rotates by θ counterclockwise. As he takes more and more steps, he will get arbitrarily close to a single point. What is the distance from this point to the origin?

10. Let $s_n = \displaystyle\sum_{a=1}^{n} \frac{1}{2^a 3^{n+1-a}}$. Then, $\displaystyle\sum_{n=1}^{\infty} \frac{s_n}{2^n} = \frac{p}{q}$ for relatively prime positive integers p, q. Find $p+q$.