RAMC 2024

Middle School Individual Round

- **SCORING:** The first 10 questions are worth 1 point each, and the last 5 questions are worth 2 points each, for a total of 20 possible points.
- This round contains 15 questions to be solved in 45 minutes. All answers are integers.
- No computational aids are permitted other than scratch paper, graph paper, and a pen/pencil. No calculators of any kind are allowed.
- Fill out your information, and sign/initial the honor code on the answer sheet provided.
- If you believe there is an error on the test, submit a challenge to the proctors. Please include your name, level (E1/E2/MS/HS), and your solution to the problem with explanation.

Do not flip the page until the proctor begins the round!

1. Evaluate $(7 \times (3 + 4 \times 5)) + 6^2$.

2. Andrea runs at a speed of 8 mph east, and Brady runs at a speed of 12 mph west. If they begin running at the same time, and meet each other after 45 minutes, how many miles apart were they originally?

3. Timothy has 5 differently sized coconuts, two of which are green. How many ways can he place his coconuts in a row such that the green coconuts are not adjacent?

4. Define the $\bigstar$ operation by $a \bigstar b = 3(a + 2b)$. If $6 \bigstar (5 \bigstar x) = 0$, what is x?

5. What is the largest 5-digit number whose digits multiply to 120?

6. Points A, B, C, and D lie on the 2D plane. If $AB = 5$, $BC = 7$, and $CD = 15$, what is the positive difference between the largest and smallest possible lengths of AD?

7. Thalia adds up all of the numbers from 1 to 100, but accidentally leaves out a number. The total she gets is a palidrome. What is the sum of all the possible numbers that Thalia might have left out?

8. Positive integers a and b satisfy $a < b$. If a has 3 positive integer factors, and b has 5, what is the smallest possible value of $b - a$?

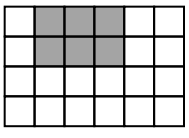
9. For digits a and b, a positive integer N has base-9 representation $\underline{a}\,\underline{0}\,\underline{b}$, and base-11 representation $\underline{b}\,\underline{0}\,\underline{a}$. What is the base-10 representation of the largest possible value of N?

10. Adam and Bailey are taking turns rolling dice. Adam has a fair 6-sided die, while Bailey has a fair 4-sided die. Adam and Bailey take turns rolling their die, starting with Adam. The probability that Adam is the first player to roll a 1 can be expressed as $\frac{m}{n}$, a fraction in simplest form. What is $m + n$?

11. Find the sum of all values of x that satisfy:

$$(x^2 - 9x + 19)^{(x^2 - 2x - 63)} = 1.$$

12. Consider a 4 by 6 grid of unit squares. How many ways are there to color a rectangle consisting of some of these unit squares such that at least 7 squares remain uncolored? One such coloring is shown to the right.



13. Points M and N lie on a circle with center O and radius 12, such that $\angle NOM = 60°$. A second circle is internally tangent to the first circle, and is also tangent to both NO and MO. Within this second circle, a square is inscribed. What is the area of the square?

14. Alice is climbing an escalator, and Bob is climbing a flight of stairs of the same length. Due to the other people on the escalator, Alice's speed, relative to the escalator steps, is only $\frac{1}{4}$ of her normal walking speed. Alice and Bob start climbing at the same time. Bob reaches the top 5 seconds before Alice, at which point Alice is 75% of the way up. Given that they have the same normal walking speed, by how many seconds would Alice beat Bob to the top if there were no other people on the escalator slowing her down?

15. The first two terms in a sequence are 202 and 4. Successive terms are calculated by taking the average of the previous two terms. As this sequence continues on for a long time, what value do its terms approach?