Rochester Area Math Competition (High School)

Hosted by Rochester Math Club

March 14th, 2018

1 Introduction

Welcome to the first annual Rochester Area Math Competition (RAMC)! RAMC is a unique opportunity for K-12 students to demonstrate their mathematical knowledge. This is the High School test; if you feel you are taking the wrong test level, please raise your hand and we will provide a different test for you.

Read through the following guidelines. Do **NOT** flip the page until you are told to do so.

1.1 Contest Format

RAMC consists of a 45 minute individual test containing 15 problems that range in difficulty. Problems at the end of the test are guaranteed to be more difficult than the first couple problems and are meant to challenge you. Problems towards the end may require trigonometry and pre-calculus ideas. Partial credit is not given. Calculators **MAY NOT** be used on this test. All answer must be filled out on the given answer sheet.

1.2 Challenges

After the contest is over, solutions will be posted on-site and on the website. If you have any challenges to test questions, please make the challenge within an hour of when the solutions are posted. On a blank sheet of paper, please include your name, grade, problem number, and your proof and explanation of why you believe your answer is (also) correct. Give your proof to one of the test mods.

1.3 Ties

Ties will be broken through a tiebreaker round. We will announce the finalist shortly after the competition. The tiebreaker round will be explained to those who make it. Awards will be determined on the day of the competition.

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2 RAMC

Information: RAMC is broken into two sections: Problems 1-10, and 11-15. Problems in the first range are each worth one point; problems in the second range are each worth two points. Partial credit is not given.

Note: The word "exact" in the questions call for an answer in simplest form. For example, instead of answering 1.73205, the right answer is $\sqrt{3}$. Please try to reasonably simply your arithmetic. Good luck!

1. Given $x = 3$, find $3x^2 + \frac{x}{3} + 5 + \frac{2}{x}$.

2. Find all values of x that satisfy $2x^2 - 13x + 21 \leq 0$.

3. A sequence is called $special$ if the next term in the sequence is the average of the two preceding terms (after the first two terms). For example, $100, 100, 100, 100...$ and $100, 50, 75, 62.5...$ are both $special$ sequences. Given that $6, 8, \cdots$ is a $special$ sequence, find the 7th term of the sequence.

4. Triangle ΔABC has sides $AB = 13$, $BC = 14$, and $CA = 15$. Point H is on BC such that $AH \perp BC$. Find AH.

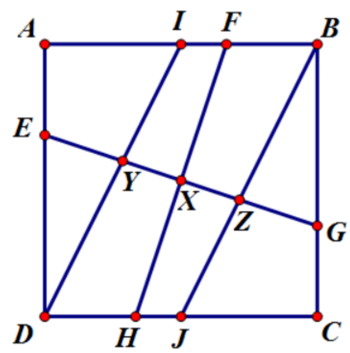
5. If $\frac{5}{x+\frac{5}{x+\frac{5}{x+\cdots}}} = \frac{\sqrt{29}-3}{2}$, determine exactly the value of x.

6. Towns A and B are 60 miles apart. Tracy and Richard start driving from A to B at the same time. They drive at constant speeds, but Tracy drives 10 mph faster than Richard does. Tracy reaches B after 1 hour and immediately starts driving back to A. How far away from A does she pass Richard on the way back?

7. If integers a and b, not necessarily distinct, are selected randomly and independently from $1 - 100$, inclusive, find the probability that $a + b$ is even.

8. Given $a + b = 3$ and $a^2 + b^2 = 12$, find the exact value of $a^6 + b^6$.

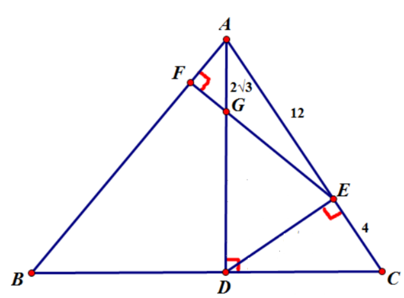
9. In square $ABCD$ with side length 6, point E is on AD such that $AE:ED = 1:2$, point F is on AB such that $BF:FA = 1:2$, point G is on BC such that $CG:GB = 1:2$, point H is on CD such that $DH:HC = 1:2$. Lines EG and HF intersect at a point X. If I and J are the midpoints of AB and CD, respectively, and DI and BJ intersect line EG at Y and Z, respectively, find $YX + XZ$.



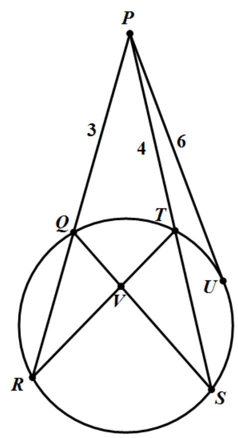
10. How many five-digit positive integers have digits that strictly increase or strictly decrease? For example, 12345 is a strictly increasing integer, but 12334 is not.

11. Six distinguishable people are sitting around a circular table, each holding a fair coin. All six people flip their coins and those who flip tails stand while those who flip heads remain seated. What is the probability that no two adjacent people will stand?

12. In triangle ABC, points D, E, F are on BC, CA, AB, respectively, such that $AD \perp BC$, $DE \perp CA$, $EF \perp AB$. AD and EF intersect at G. Given that $CE = 4$, $EA = 12$, and $AG = 2\sqrt{3}$, find EG.



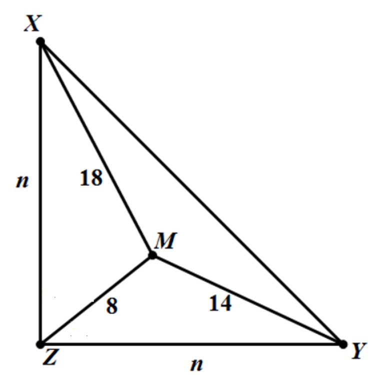
13. In the figure below (not drawn to scale), points Q, R, S, and T lie on the circle. If PU is tangent to the circle and has length 6, $PQ = 3$ and $PT = 4$, and that points P, Q, R are collinear and P, T, S are collinear, determine exactly the ratio of the area of quadrilateral $PQVT$ to the area of ΔQRV.



14. The roots of the polynomial $x^3 + 4x - 1 = 0$ are r_1, r_2 and r_3. Find exactly

$$\frac{2r_1^2}{(3r_2+1)(3r_3+1)} + \frac{2r_2^2}{(3r_1+1)(3r_3+1)} + \frac{2r_3^2}{(3r_1+1)(3r_2+1)}$$

15. Isosceles $\triangle XYZ$ has a right angle at Z. Point M is inside $\triangle XYZ$, such that $MX = 18$, $MY = 14$, and $MZ = 8$. If legs $\overline{XZ}$ and $\overline{YZ}$ have length n, find the exact value of n.



This is the end of the test. Please go back and check your answers until your time is up.