Rochester Area Math Competition (Middle School)

Hosted by Rochester Math Club

March 14th, 2018

1 Introduction

Welcome to the first annual Rochester Area Math Competition (RAMC)! RAMC is a unique opportunity for K-12 students to demonstrate their mathematical knowledge. This is the Middle School test; if you feel you are taking the wrong test level, please raise your hand and we will provide a different test for you.

Read through the following guidelines. Do **NOT** flip the page until you are told to do so.

1.1 Contest Format

RAMC consists of a 45-minute individual test containing 15 problems that range in difficulty. Problems at the end of the test are guaranteed to be more difficult than the first couple problems and are meant to challenge you. Partial credit is not given.Calculators **MAY NOT** be used on this test. All answer must be filled out on the given answer sheet.

1.2 Challenges

After the contest is over, solutions will be posted on-site and on the website. If you have any challenges to test questions, please make the challenge within an hour of when the solutions are posted. On a blank sheet of paper, please include your name, grade, problem number, and your proof and explanation of why you believe your answer is (also) correct. Give your proof to one of the test mods.

1.3 Ties

Ties will be broken through a tiebreaker round. We will announce the finalist shortly after the competition. The tiebreaker round will be explained to those who make it. Awards will be determined on the day of the competition.

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2 RAMC

Information: RAMC is broken into two sections: Problems 1-10, 11-15. Problems in the first range are each worth one point; problems in the second range are each worth two points. Partial credit is not given.

Note: The word "exact" in the questions call for an answer in simplest form. For example, instead of answering 1.73205, the right answer is $\sqrt{3}$. Please try to reasonably simply your arithmetic. Good luck!

1. Solve the equation $4(7^2 + x) = 28 - 12x$.

2. In the arithmetic sequence 107, 88, 69, ..., what is the 7th term?

3. If the degree measures of the angles of a triangle are in the ratio 2 : 3 : 5, what is the degree measure of the largest angle of the triangle?

4. Tracy goes to a friend's house. Her friend has some cats and some chickens. Tracy counts that there are 11 animals in total (not including Tracy and her friend), and they have 28 legs in total. Each cat has 4 legs, and each chicken has 2 legs. What is the positive difference between the number of cats and the number of chickens Tracy's friend has?

5. A triangle has side lengths 9, 40, and 41. Find the shortest altitude in this triangle.

6. Consider a parallelogram in the coordinate plane. The coordinates of three of its vertices are $(0,0)$, $(3,7)$, and $(4,5)$. The fourth vertex is at (x, y). Find the sum of all possible values of x.

7. If there are 5 identical balls and 3 identical boxes, how many ways can you put the balls in the boxes if you can leave boxes empty, but all balls have to be stored?

8. How many positive 3-digit integers satisfy the condition that the product of their digits is 72?

9. A cube is inscribed in a sphere of radius 2, and another sphere is inscribed in the cube. What is radius of the smaller sphere?

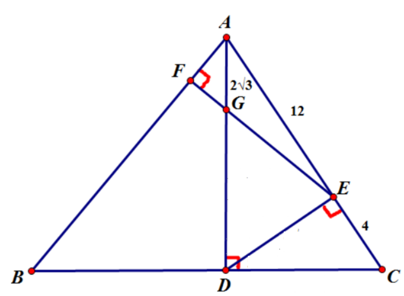
10. If 363 in base b is equal to 243 in base 10, what is b?

11. Find the remainder when the last two digits of 11^9 are divided by 6.

12. Richard and Tracy are playing a dice game. If a player rolls $1-5$, it changes to the other player's turn. If a player rolls a 6, s/he rolls again. The first person to roll two 6's in a row in his/her turn wins. If Richard goes first in this game, what is the probability that he wins with no prior 6's rolled by him or Tracy?

13. Given $a+b=3$ and $a^2+b^2=12$, find exactly the value of a^6+b^6.

14. In triangle ABC, points D,E,F are on BC,CA,AB, respectively, such that $AD \perp BC$, $DE \perp CA$, $EF \perp AB$. AD and EF intersect at G. Given that $CE=4$, $EA=12$, and $AG=2\sqrt{3}$, find EG.



15. The roots of the polynomial $x^3+4x-1=0$ are r_1, r_2 and r_3. Find exactly

$$\frac{2r_1^2}{(3r_2+1)(3r_3+1)}+\frac{2r_2^2}{(3r_1+1)(3r_3+1)}+\frac{2r_3^2}{(3r_1+1)(3r_2+1)}$$

This is the end of the test. Please go back and check your answers until your time is up.